Summation Invariant Features for 3D Face Recognition

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***Abstract*— A novel summation invariant feature under transformation group action for 3D surface recognition is proposed, and its application to 3D face recognition is investigated. Based on a systematic mathematical procedure called moving frame, we derived the summation invariant feature that is invariant under affine transformation. Compared with classical differential invariants, such as the mean curvature or the Gaussian curvature, summation invariant feature is far less sensitive to observation noise in the data. A further enhancement leads to a new type of invariant 3D surface shape descriptor called a semi-local summation invariant. We demonstrate one important, potential application of this new feature to 3D human face recognition.**

I. Introduction

Invariants for transformation groups play an important role in computer vision. The idea that one can compute functions of images that do not change under various viewing conditions is appealing for many applications such as human face recognition. Hence the study of invariants for certain transformation groups (Euclidean, affine and projective) has flourished in recent years. However, for practical applications, the shapes or 3D surfaces of the object of interests are often corrupted by noise due to imperfect data acquisition, quantization, and other causes. It has been reported that differential invariants depending on derivatives are very sensitive to noise [1], [2]. Toward the end of the last millennium, algorithms based on invariants did not meet our expectations.

There have been several attempts to decrease sensitivity to noise. To avoid high-order derivatives, a semi-differential invariant was introduced in [3], [4]. Potentials were used as coordinates to prolong group actions, so that the resulting invariants would depend on integrals rather than derivatives and not be sensitive to noise [5]. Also another type of integral invariant was formulated by integrating with respect to affine quasi-invariant arc-length [6]. These invariants are defined on continuous functions. When applied to digitized object descriptions of contours or surfaces, numerical integration will be needed and the results can be affected significantly by step size and other detailed settings.

Recently [7], we introduce a general method to generate invariants that are weighted summations of discrete data, as analogues to integral ones. Since these invariants are defined explicitly on discrete data, they do not require computationally intensive numerical integration to compute and will not be affected by the choice of step size. On the other hand, using weighted summation to compute the summation invariant will greatly reduce the impact of noise and hence promises higher signal to noise ratio of the computed invariant features. Specifically, in [7], we proposed a semi-local summation invariant feature for two-dimensional (2D) closed contours, and has demonstrated the superior performance using this novel summation invariant feature compared to those produced using integral invariants or wavelet invariant features. We note that the 2D summation invariant formulation is quite different from those developed in [5].

In this paper, we focus on deriving a novel summation invariant formulation for 3D surface. We note that the integral invariant proposed by Hann and Hickman [5] can not be easily extended to 3D objects even the shape of the surface is described by equations.

In order to apply the proposed summation invariants to object recognition, in particular face recognition problems, additional challenges must be overcome. Specifically, the 3D summation invariant maps a high dimension 3D surface into a scalar. As such, it may not yield sufficient amount of information to distinguish similar but different faces. In order to enhance the discriminating power of this invariant feature, we propose to partition a given 3D surface into smaller non-overlapping patches using conformal mapping and compute summation invariant on each patch. Preliminary results indicate such an approach is very promising.

The rest of this paper is organized as follows. Section 2 describes the summation invariant. In section 3, we use the summation invariant to define a novel shape descriptor, which is called the semi-local summation invariant. In section 4, we test the proposed method on 3D mesh under translation and rotation. Finally, section 5 summarizes the contribution and provides an overview of future directions.

II. Summation Invariant

The transformation groups acting on $\mathbb{R}^3$, such as the Euclidean and affine groups are of particular importance in 3D object recognition. In this section, we describe a systematic method to find summation invariant for surface in 3D space.

A. Extending Group Action to Potentials

Hann and Hickman [5] defined a potential jet space for a transformation group acting on $\mathbb{R}^2$. Based on this definition, they have derived *integral invariants* for 2D contours. A potential drawback of applying their method for practical contour recognition is that numerical integration will be needed to evaluate the integral invariant since practical contours are always represented by sampled 2D coordinates. In [7], we derived a 2D summation invariant feature and illustrated with an example to show that such a new feature performs better than direct evaluation of Hann and Hickman's integral invariant features using numerical integration.

In this paper, we will focus on 3D invariant features that represent a 3D surface of an object, such as a human face. Unfortunately, the direct generalization of Hann and Hickman's 2D integral invariant approach for 3D surface objects is quite a non-trivial task. Instead, in this paper, we employ a somewhat different representation of a given 3D *surface*. Consider a surface $S \subseteq \mathbb{R}^3$; we can treat it as a mapping from a simple closed region $\mathcal{U} \subseteq \mathbb{R}^2$ to $\mathbb{R}^3$

$$S : (u, v) \longmapsto (x, y, z), \text{where}(u, v) \in \mathcal{U} \tag{1}$$

In many engineering fields, we only have values of a surface S measured at discrete coordinates Hence, S can be represented as a discrete function with two independent variables.

$$S = \begin{bmatrix} x[m,n] \\ y[m,n] \\ z[m,n] \end{bmatrix} \tag{2}$$

where $m = 1, \dots, M$ and $n = 1, \dots, N$. Based on this assumption, the *potential* and *potential jet space* can be defined as follows:

Definition 1: The potential $P_{i,j,k}$ of order l is given by

$$P_{i,j,k} = \sum_{m=1}^{M} \sum_{n=1}^{N} x^i[m,n] \cdot y^j[m,n] \cdot z^k[m,n] \tag{3}$$

where $i + j + k = l$.

Definition 2: The potential jet space J^l is the Euclidean space with coordinates

$$\begin{aligned} J^l =& (x[1,1], y[1,1], z[1,1], x[M,1], y[M,1], \\ & z[M,1], x[1,N], y[1,N], z[1,N], P^{(l)}) \end{aligned}$$

where $P^{(l)}$ consists of potentials up to l^{th} order.

Then, invariant functions of the transformation group G can be found by the *method of moving frames* [1].

B. Invariants of Affine Transformations

A surface under affine transformation can be described as

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} j \\ k \\ L \end{bmatrix} = \begin{bmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{bmatrix} \tag{4}$$

To systematically derive the invariants, we apply the *method of moving frames* formulated by Élie Cartan [8]:

1) First, the group action is prolonged into a jet space which is spanned by the (x, y, z) coordinates as well as their corresponding directives. We may solve for group parameters using *normalization equations*.
2) Then, these group parameters will be substituted into un-normalized jet space coordinates and that will yield the desired differential invariants.

Now, we will derive affine invariants by using this method and also explain some terminologies used above. *Prolongation* of group action is simply applying the affine transformation to those coordinates defined by potentials. It's a prolonged group action since the transformation group originally acts on $\mathbf{R}^3$. After prolonging the affine transformation to $P_{1,0,0}$, it becomes

$$\begin{aligned} \overline{P}_{1,0,0} &= \sum_{m=1}^{M} \sum_{n=1}^{N} \overline{x}[m,n] \\ &= \sum_{m=1}^{M} \sum_{n=1}^{N} (ax + by + cz + j) \\ &= aP_{1,0,0} + bP_{0,1,0} + cP_{0,0,1} + jMN \end{aligned}$$

$\overline{P}_{0,1,0}$ and $\overline{P}_{0,0,1}$ can be found in the same way. We can solve for group parameters $\{a, b, c, d, e, f, g, h, i, j, k, L\}$ by setting a *normalization equation* as follows:

$$\begin{aligned} &(\overline{x}[1,1], \overline{y}[1,1], \overline{z}[1,1], \overline{x}[M,1], \overline{y}[M,1], \overline{z}[M,1], \\ &\overline{x}[1,N], \overline{y}[1,N], \overline{z}[1,N], \overline{P}_{1,0,0}, \overline{P}_{0,1,0}, \overline{P}_{0,0,1}) \\ &= (0,0,0,1,0,0,0,1,0,0,0,0) \end{aligned}$$

The solved $\{a, b, c, d, e, f, g, h, i, j, k, L\}$ is called a *moving frame* [1], [2]. It yields an affine transformation which brings any point in *potential jet space* to a fixed point in *potential jet space*. In this case, we choose the fixed point as $(0,0,0,1,0,0,0,1,0,0,0,0)$. In fact, we are free to choose any fixed point in potential jet space as long as we can solve for $\{a, b, c, d, e, f, g, h, i, j, k, L\}$. By applying this *moving frame* to higher order affine-transformed potentials, we can generate as many invariants as we want. For example, an affine invariant $\eta_{2,0,0}$ of the surface, which is shown below, can be found by substituting $\{a, b, c, d, e, f, g, h, i, j, k, L\}$ into $\overline{P}_{2,0,0}$.

$$\begin{aligned} \eta_{2,0,0} = \{&P_{0,0,2}(MN(x_{01}y_{00} - x_{00}y_{01}) + P_{1,0,0}(y_{01} - y_{00}) \\ &+ P_{0,1,0}(x_{00} - x_{01}))^2 - 2P_{0,1,1}(MN(x_{01}y_{00} - x_{00}y_{01}) \\ &+ P_{1,0,0}(y_{01} - y_{00}) + P_{0,1,0}(x_{00} - x_{01}))(MN(x_{01}z_{00} \\ &- x_{00}z_{01}) + P_{1,0,0}(z_{01} - z_{00}) + P_{0,0,1}(x_{00} - x_{01})) \\ &+ P_{0,2,0}(MN(x_{01}z_{00} - x_{00}z_{01}) + P_{1,0,0}(z_{01} - z_{00}) \\ &+ P_{0,0,1}(x_{00} - x_{01}))^2 - 2P_{1,1,0}(MN(x_{01}z_{00} - x_{00}z_{01}) \\ &+ P_{1,0,0}(z_{01} - z_{00}) + P_{0,0,1}(x_{00} - x_{01}))(MN(y_{01}z_{00} \\ &- y_{00}z_{01}) + P_{0,1,0}(z_{01} - z_{00}) + P_{0,0,1}(y_{00} - y_{01})) \\ &+ P_{2,0,0}(MN(y_{01}z_{00} - y_{00}z_{01}) + P_{0,1,0}(z_{01} - z_{00}) \end{aligned}$$

$$+ P_{0,0,1}(y_{00} - y_{01}))^2 - 2P_{1,0,1}(MN(x_{01}y_{00} - x_{00}y_{01})$$
$$+ P_{1,0,0}(y_{01} - y_{00}) + P_{0,1,0}(x_{00} - x_{01}))(MN(y_{00}z_{01}$$
$$- y_{01}z_{00}) + P_{0,1,0}(z_{00} - z_{01}) + P_{0,0,1}(y_{01} - y_{00}))$$
$$- MN(P_{1,0,0}(y_{01}z_{00} - y_{00}z_{01}) + P_{0,1,0}(x_{00}z_{01} - x_{01}z_{00})$$
$$+ P_{0,0,1}(x_{01}y_{00} - x_{00}y_{01}))^2\}$$
$$/(MN(x_{00}(y_{10}z_{01} - y_{01}z_{10}) + x_{10}(y_{01}z_{00} - y_{00}z_{01})$$
$$+ x_{01}(y_{00}z_{10} - y_{10}z_{00}))$$
$$+ P_{1,0,0}(y_{00}(z_{01} - z_{10}) + y_{10}(z_{00} - z_{01}) + y_{01}(z_{10} - z_{00}))$$
$$+ P_{0,1,0}(x_{00}(z_{10} - z_{01}) + x_{10}(z_{01} - z_{00}) + x_{01}(z_{00} - z_{10}))$$
$$+ P_{0,0,1}(x_{00}(y_{01} - y_{10}) + x_{10}(y_{00} - y_{01}) + x_{01}(y_{10} - y_{00})))^2$$

where $x_{00} = x[1,1], y_{00} = y[1,1], z_{00} = z[1,1], x_{10} = x[M,1], y_{10} = y[M,1], z_{10} = z[M,1], x_{01} = x[1,N], y_{01} = y[1,N], z_{01} = z[1,N]$. Note that summation invariant is quite different from the traditional moment invariants [9]. The moment invariants are defined globally, i.e. the whole shape is required. On the contrary, summation invariants can be defined on any region of a surface such that the local feature can be extracted. In order to distinguish similar objects, an invariant function which can represent local characteristics of an object is always highly desired. It can be shown that the numerator and denominator of η_{200} are respectively invariant to Eucledean transform.

III. Applications to 3D Face Recognition

There are a number of efforts in the past to explore 3D invariant features for the purpose of human face recognition using local invariant features such as gradients [10], or view based approach [11]. To apply the summation invariant features presented above to face recognition, a number of practical issues must be addressed.

In practical applications, the summation invariant can be computed locally over a patch of surface to extract regional features. We call it the *semi-local summation invariant*. By controlling the size of the surface, both unique local features as well as global features may be captured.

Another practical consideration is that 3D surfaces are often represented by a triangular mesh which has no ordering information among points on the surface. However, the summation invariant as well as semi-local summation invariants are defined over regular grid points in a 3D space. We apply a *shape-preserving parameterization* [12] method to convert triangular mesh representation to regular grid representation.

Our plan for 3D face recognition is as follows: For a given surface representing human face, we partition the surface into patches of regular rectangle support in the parameterized space. Then we compute the semi-local summation invariants over each of the surface patch. This yields a feature matrix with each element representing the semi-local summation invariant computed over the corresponding surface patch. By comparison a distance between these feature matrices of two surfaces, one would be able to determine whether two surfaces are similar.

To illustrate, we conduct a preliminary experiment as follows: We down-loaded triangular meshes of human faces from the model library at 3D Cafe [13]. Then we apply Euclidean transformation of a human surface. These two meshes are parameterized using shape-preserving mapping [12] with 100 sampling points in both longitude and latitude directions. The results of surface parameterization are shown in Figure 1. We further divide the parameterized surface into 10×10 disjoint regions whose boundaries are shown in solid lines in Figure 1. Then, semi-local summation invariants, *numerator* of the $\eta_{2,0,0}$, are computed for each region of these two parameterized facial surfaces. This yields a 10×10 feature matrix for each of these surfaces as shown in Figure 2. Clearly, the two feature matrices so computed are identical, illustrating the invariance of the proposed summation invariant feature with respect to Euclidean transformation (rotation and translation).

IV. Conclusion

In this work, a transformation group action on $\mathbb{R}^3$ is extended to jet space defined by potentials, which are summations of (x, y, z) coordinates. Thus, we provide a new solution to the equivalence problem of surfaces in 3D space under transformation group action. A summation invariant for the affine group acting on $\mathbb{R}^3$ is explicitly derived. The advantage of using summation invariants is that they are less sensitive to noise because they do not depend on derivatives. The resulting feature vector for object recognition, therefore, will be much more reliable. Preliminary experiments on triangular meshes clearly indicate that the proposed invariant function has the potential to be practically applied to 3D object recognition.

Despite these encouraging results, the method has been only tested on synthesized data. We would like to apply it to a real life problem and attempt to recognize human faces under rotation and translation.

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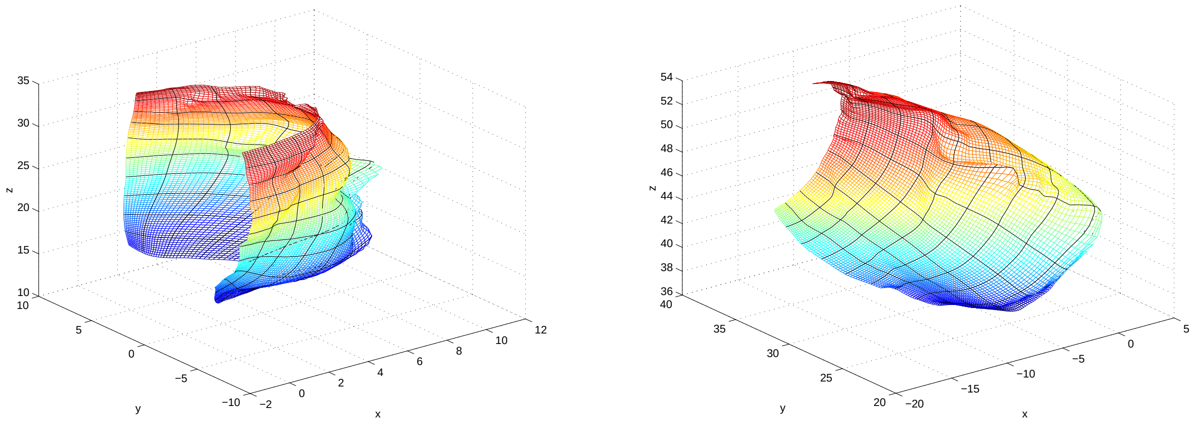
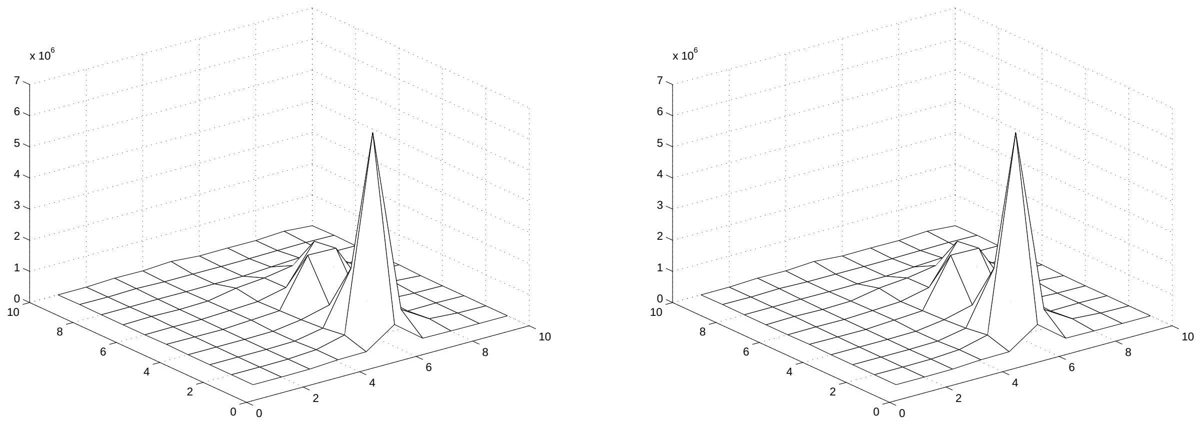


(a) Parameterization of original mesh (b) Parameterization of transformed mesh

Fig. 1. Parameterization of triangular meshes: The black lines divide face into 10×10 disjoint regions and summation invariants are computed at each region.



(a) numerator($\lambda_{2,0,0}$) of of original mesh (b) numerator($\eta_{2,0,0}$) of transformed mesh

Fig. 2. Summation invariants computed by using numerator(η_{200}).

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