Numerical Analysis
Quiz 5: Interpolation and Numerical Integration

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June 20, 2007

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1. (50 %) Glycerin is a liquid used in the manufacturing of many products. The viscosity of glycerin is a function of temperature, as the data shown in the following table.

T	0	10	20	30	40	50
μ	10.60	3.810	1.492	0.629	0.2754	0.1867

Please estimate the viscosity of glycerin at 22°C by quadratically interpolating the data points at $T_1 = 30$, $T_2 = 40$, and $T_3 = 50$. Compare the results to the interpolant using $T_1 = 10$, $T_2 = 20$, and $T_3 = 30$.

Answer:

$$\mu(T) = f[T_1] + f[T_1, T_2](T - 30) + f[T_1, T_2, T_3](T - 30)(T - 40)$$

$T = 22$, $T_1 = 30$, $T_2 = 40$, $T_3 = 50$

$$f[T_1] = 0.629$$

$$f[T_1, T_2] = \frac{0.2754 - 0.629}{40 - 30} = -0.03536$$

$$f[T_1, T_2, T_3] = \frac{f[T_1, T_2] - f[T_1, T_3]}{T_3 - T_2} = \frac{-0.022115 + 0.03536}{50 - 40}$$

$$\mu(22) = 0.629 + (-0.03536) \times (22 - 30) + 0.0013245 \times (22 - 30) \times (22 - 40) = 1.102608$$

2. (50 %) The second order Lagrange interpolating polynomial passing through three points can be written as

$$P_2(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}f_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}f_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}f_3$$

Using $P_2(x)$ as an approximation to $f(x)$, the integral of $f(x)$ is approximately

$$I = \int_{x_1}^{x_3} f(x)dx \approx \int_{x_1}^{x_3} P_2(x)dx = \frac{h}{3}\Big(f_1 + 4f_2 + f_3\Big) \tag{1}$$

Please show all the intermediate steps in Equation (??).

Answer:

$$h = \frac{x_3 - x_1}{2} \ , \ x_2 - x_1 = h \ , \ x_3 - x_2 = h$$

$$\begin{aligned}
& \int_{x_1}^{x_3} P_2(x)dx \\
= \ & \int_{x_1}^{x_3} \left(\frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}f_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}f_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}f_3 \right) dx \\
= \ & \int_{x_1}^{x_3} \left(\frac{(x-x_2)(x-x_3)}{(-h)(-2h)}f_1 + \frac{(x-x_1)(x-x_3)}{(h)(-h)}f_2 + \frac{(x-x_1)(x-x_2)}{(2h)(h)}f_3 \right) dx \\
= \ & \frac{1}{2h^2}\int_{x_1}^{x_3} (x^2 + (-x_2 - x_3)x + x_2x_3)f_1dx \\
& + \frac{1}{-h^2}\int_{x_1}^{x_3} (x^2 + (-x_1 - x_3)x + x_1x_3)f_2dx \\
& + \frac{1}{2h^2}\int_{x_1}^{x_3} (x^2 + (-x_1 - x_2)x + x_1x_2)f_3dx \\
= \ & \ldots\ldots \\
= \ & \frac{h}{3}(f_1 + 4f_2 + f_3)
\end{aligned}$$