Numerical Analysis
Quiz 3: Solving Systems of Equations

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1. (50 %) Write a MATLAB function to solve $\mathbf{Ax} = \mathbf{b}$ when $\mathbf{A}$ is a lower triangular matrix.

   Answer:

```
function x = Islove(A,b)
[m,n] = size(A);
if m ~= n
    error('A matrix is not square');
end
x = zeros(n,1);
x(1) = b(1)/A(1,1);
for i = 2:n
    x(i)=(b(i) - A(i , 1:i-1) * x(1:i-1))/A(i,i);
end
```

2. (50 %) Manually solve $\mathbf{QRx} = \mathbf{b}$ for $\mathbf{x}$, where

$$\mathbf{Q} = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}, \quad \mathbf{R} = \sqrt{2}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 2\sqrt{2} \\ 4 \end{bmatrix}.$$

(*Hint*: Take advantage of the fact that $\mathbf{QQ}^T = \mathbf{Q}^T\mathbf{Q} = \mathbf{I}$.)

Answer:

$$\begin{aligned} & \mathbf{QRx} = \mathbf{b} \\ \Rightarrow \quad & \mathbf{Q}^T\mathbf{QRx} = \mathbf{Q}^T\mathbf{b} \\ \Rightarrow \quad & \mathbf{Rx} = \mathbf{Q}^T\mathbf{b} \text{ (because of the } \textit{Hint}.) \end{aligned}$$

$$\mathbf{Rx} = \mathbf{Q}^T\mathbf{b}$$

$$\Rightarrow \sqrt{2}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 2 \\ 2\sqrt{2} \\ 4 \end{bmatrix} = \begin{bmatrix} 3\sqrt{2} \\ 2\sqrt{2} \\ \sqrt{2} \end{bmatrix}$$

$$\Rightarrow \begin{cases} \sqrt{2}(x_1 + x_2 + x_3) = 3\sqrt{2} \\ \sqrt{2}(x_2 + x_3) = 2\sqrt{2} \\ \sqrt{2}x_3 = \sqrt{2} \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \end{cases}$$

$$\Rightarrow x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$