Numerical Analysis
Quiz 2: Finding the Roots of $f(x) = 0$

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1. (50 %) By using Newton's method, please derive the *iteration function* $g(x)$ for finding the roots of $x - x^{1/3} - 2 = 0$.

   Answer:

   The function $f(x)$ and its derivative are

   $$f(x) = x - x^{1/3} - 2 \qquad f'(x) = 1 - \frac{1}{3}x^{-2/3}$$

   The corresponding iteration function is given by

   $$g(x) = x - \frac{f(x)}{f'(x)}$$

   or

   $$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

2. (50 %) The function $f(x) = sin(x^2) + x^2 - 2x - 0.09$ has four roots in the interval $-1 \leq x \leq 3$. Given the m-file `fx.m`, which contains

```
function f = fx(x)
f = sin(x.^2) + x.^2 - 2*x - 0.09;
```

the statement

```
>> brackPlot('fx',-1,3)
```

produces only two brackets. What needs to be changed so that all four roots are found?

Note that the bracketing algorithm is as follows.

given: $f(x), x_{min}, x_{max}, n$

$dx = (x_{max} - x_{min})/n$
$a = x_{min}$
$i = 0$

while $i < n$
  $i \leftarrow i + 1$
  $b = a + dx$
  if $f(x)$ changes sign in [a,b]
    save [a,b] for root finding
  end
  a = b
end

Answer:

The two roots are missed by `brackPlot` because the default search interval is too coarse. All four roots could be found by using a finer search interval.