Numerical Analysis

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Name : ______________ **Student ID** : __________________

Midterm Examination
Wednesday, April 25, 2007
from 2:45 to 4:00 PM

1. (20 %) Translate Algorithm 5.1 which is shown below into a Matlab function that computes the bit pattern of a floating-point mantissa.

Algorithm 5.1 Conversion from Floating-Point to Binary

$r_0 = x$
for $k = 1, 2, \ldots, m$
  if $r_{k-1} \geq 2^{-k}$
    $b_k = 1$
    $r_k = r_{k-1} - 2^{-k}$
  else
    $b_k = 0$
  end if
end for

Answer:

```
b = zeros(1,m);
r = x;
for k = 1:m
   if r > 2^(-1*k)
     b(k) = 1;
     r = r - 2^(-1*k);
   else
     b(k) = 0;
   end
end
```

2. (20 %) What is the difference between `realmax` and the first double-precision floating-point number less than `realmax`?

Answer:

The largest double precision floating-point number is

$$(2^{-1} + 2^{-2} + \ldots + 2^{-51} + 2^{-52}) \times 10^{1023}$$

because there are 53 bits allocated to the mantissa and 11 bits allocated to the exponent. Similarly, the second largest double precision floating-point number is

$$(2^{-1} + 2^{-2} + \ldots + 2^{-51}) \times 10^{1023}$$

Therefore, the difference between them is

$$2^{-52} \times 10^{1023}$$

3. (20 %) Given the function $f(x) = (1-x)^{-1}$, please derive the Taylor series approximations $P_1(x)$, $P_2(x)$, and $P_3(x)$ to $f(x)$ at $x = x_0$.

Answer:

The Taylor series approximations to $f(x)$ of order 1, 2 and 3 are

$$
\begin{aligned}
P_1(x) &= \frac{1}{1-x_0} + \frac{x-x_0}{(1-x_0)^2} \\
P_2(x) &= \frac{1}{1-x_0} + \frac{x-x_0}{(1-x_0)^2} + \frac{(x-x_0)^2}{(1-x_0)^3} \\
P_3(x) &= \frac{1}{1-x_0} + \frac{x-x_0}{(1-x_0)^2} + \frac{(x-x_0)^2}{(1-x_0)^3} + \frac{(x-x_0)^3}{(1-x_0)^4}
\end{aligned}
$$

4. (20 %) David Peters obtains the following equation for the optimum damping ratio of a spring-mass-damper system designed to minimize the transmitted force when an impact is applied to the mass:

$$cos\left[4\xi\sqrt{1-\xi^2}\right] = -1 + 8\xi^2 - 8\xi^4$$

Please write a MATLAB function for finding the roots of this equation with the secant method.

Answer:

```
function y = f(x)
y = cos( 4*x*sqrt(1 - x^2)) + 1 - 8*x^2 + 8*x^4;

function r = second(x0, x1)
maxit = 100;
for k = 1:maxit
   r = x1 - f(x1)*(x1 - x0)/(f(x1) - f(x0));
   if CONVERGENCE
     break;
   end
   x0 = x1;
   x1 = r;
end
```

5. (20 %) Derive an iterative formula for finding the roots of $\cos(x) = x$ with Newton's method.

Answer:

The function $f(x)$ and its derivative are

$$f(x) = cos(x) - x \qquad f'(x) = -sin(x) - 1$$

The corresponding iteration function is given by

$$g(x) = x - \frac{f(x)}{f'(x)}$$

or

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$