Image Registration Techniques
Homework 2
Due: Wednesday July 15th at the start of class

This homework explores Lectures 2 and 3. Answer each of the following questions clearly. You may submit hand-written answers, but if your handwriting isn't clear, please typeset your solutions.

1. (**5 points**) Given two vectors $\mathbf{v}$ and $\mathbf{u}$. Find the two components of $\mathbf{v}$, one in the direction of $\mathbf{u}$ and one perpendicular to $\mathbf{u}$.

2. (**10 points**) Show that if $\|\mathbf{Av}\| = \|\mathbf{v}\|$ for all vectors $\mathbf{v}$ then $\mathbf{A}$ must be orthonormal.

3. (**10 points**) Find the inverse of the mapping from physical to pixel coordinates. Write it using matrix and vector notation in both homogeneous and non-homogeneous forms.

4. (**5 points**) Re-examine the algebraic formulation of the 2d similarity transformation from Lecture 3. Write parameters s and α as a function of a and b and write parameters a and b in terms of s and α. This will show that you can easily move back-and-forth between representations.

5. (**5 points**) Find the inverse of the similarity transformation,

$$\mathbf{x}' = s\mathbf{R}\mathbf{x} + \mathbf{t}$$

   and show that it is a similarity transformation. (You need only show that it has the form of a similarity transformation.)

6. (**10 points**) Prove that the eigenvectors associated with distinct eigenvalues of a symmetric matrix are orthogonal. **Hint:** let $\mathbf{A}$ be a symmetric matrix, let λ_1 and λ_2 be eigenvalues with $\lambda_1 \neq \lambda_2$, and let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the associated eigenvectors; then consider the expression $\mathbf{v}_1^T \mathbf{A} \mathbf{v}_2$.