Course: Introduction to Image Processing
Homework: #4
Due date: 5/04, 11:59pm

This is individual work. However, peer discussion is encouraged. For every programming assignment, clearly document your code so that the TA can follow your algorithms easily. Include the output images of your functions in the report.

Question 1. We define the 6-neighbors of a pixel p with coordinate (x,y) to be the pixels with coordinates (x+1,y), (x-1,y), (x,y+1), (x,y-1), (x+1, y+1) and (x-1, y-1).
   a. Find the lengths of the shortest 6-path between pixels with the following coordinates:
      - (2,1) and (6,8)
      - (3,1) and (7,4)
      - (4,1) and (2,5)
      - (2,5) and (6,2)
   b. Develop an expression for 6-path metric, \( d_6(x,y) \).
   c. Refine the algorithm for component labeling presented in lecture12 (starting from slide 11) to label the 6-components of a binary image.
   d. For the following image use your algorithm to label the 6-components.

Question 2. Let A denote the set shown shaded in the following figure. Refer to the structuring elements shown (the black dots denote the origin). Sketch the result of the following morphological operators:
   a. \( (A \ominus B^4) \oplus B^2 \)
   b. \( (A \ominus B^1) \oplus B^3 \)
Question 3. A preprocessing step in an application of microscopy is concerned with the issue of isolating individual round particles from similar particles that overlap in groups of two or more particles. Assuming that all particles are of the same size and the area is known as $R$, propose a morphological algorithm that produces 3 images consisting respectively of

a. Only of particles that have merged with the boundary of the image
b. Only overlapping particles
c. Only non-overlapping particles

Hint1: Make good use of the concept of connected components.
Hint2: You may alter the image temporarily for processing if that helps.
Question 4. The center rectangle in the binary image shown below is of size $n \times m$ pixels.

a. Sketch $G_x$ and $G_y$ and gradient magnitude ($\approx |G_x| + |G_y|$) of this image.

Assume that $G_x$ and $G_y$ are obtained using the Sobel operators. Show all relevant different pixel values in the gradient images.

b. Sketch the histogram of edge directions (which is perpendicular to the gradient direction). Be precise in labeling the height of each peak of the histogram.

c. Sketch the Laplacian of the image for the approximation filter $h_2$ in lecture5-slide13. Show all relevant different pixel values in the Laplacian image.