FD VI: Fuzzy Screening Systems

We describe a procedure, which we call the *Fuzzy Screening* method. This procedure is useful in environments in which we must select, from a large class of alternatives, a small subset to be further investigated.

This initial screening procedure is based on preliminary information. The technique suggested here requires only a non-numerical scale for this the evaluation and selection of alternatives.

Using this procedure each alternative is evaluated by each expert for satisfaction to his multi-criteria selection function.

Each criteria can have a different degree of importance. The individual expert evaluations can then be aggregated to obtain an overall evaluation function.

- R.R.Yager, Fuzzy Screaning Systems, in: R.Lowen and M.Roubens eds., *Fuzzy Logic: State of the Art* (Kluwer, Dordrecht, 1993) 251-261.

In *screening* problems one usually starts with a large subset, X, of possible alternative solutions.

Each alternative is essentially represented by a minimal amount of information supporting its appropriateness as the best solution.

This minimal amount of information provided by each alternative is used to help select a subset A of X to be further investigated.

Two prototypical examples of this kind of problem can be mentioned.

- **Job selection problem.** Here a large number of candidates, X, submit a resume, minimal information, to a job announcement. Based upon these resumes a small subset of X, A, are called in for interviews. These interviews, which provide more detailed information, are the basis of selecting winning candidate from A.
- **Proposal selection problem**. Here a large class of candidates, X, submit preliminary proposals, minimal information. Based upon these preliminary proposals a small subset of X, A, are requested to submit full detailed proposals. These detailed proposals are the basis of selecting winning candidate from A.

In the above examples the process of selecting the subset A, required to provide further information, is called a *screening process*.

Yager suggests a technique, called *fuzzy screening system*, for managing this screening process.

This kinds of screening problems described above besides being characterized as decision making with minimal information general involve multiple participants in the selection process. The people whose opinion must be considered in the selection process are called *experts*.

Thus screening problems are a class of multiple expert decision problems.

In addition each individual expert's decision is based upon the use of multiple criteria. So we have ME-MCDM

(**M**ulti **E**xpert-**M**ulti **C**riteria **D**ecision **M**aking)

problem with minimal information.

The fact that we have minimal information associated with each of the alternatives complicates the problem because it limits the operations which can be performed in the aggregation processes needed to combine the multi-experts as well as multi-criteria.

The Arrow impossibility theorem

- Arr51 K.J. Arrow, *Social Choice and Individual Values* (John Wiley & Sons, New York, 1951).

is a reflection of this difficulty.

Yager suggests an approach to the screening problem which allows for the requisite aggregations but which respects the lack of detail provided by the information associated with each alternative. The technique only requires that preference information be expressed in by elements draw from a scale that essentially only requires a linear ordering. This property allows the experts to provide information about satisfactions in the form of a linguistic values such as *high, medium, low*. This ability to perform the necessary operations will only requiring imprecise linguistic preference valuations will enable the experts to comfortably use the kinds of minimally informative sources of information about the objects described above. The fuzzy screening system is a two stage process.

- In the first stage, individual experts are asked to provide an evaluation of the alternatives. This evaluation consists of a rating for each alternative on each of the criteria.
- In the second stage, we aggregate the individual experts evaluations to obtain an overall linguistic value for each object. This overall evaluation can then be used by the decision maker as an aid in the selection process.

The problem consists of three components.

- The first component is a collection

  $$X = \{X_1, \dots, X_p\}$$

  of alternative solutions from amongst which we desire to select some subset to be investigated further.
- The second component is a group

  $$A = \{A_1, \dots, A_r\}$$

  of experts or panelists whose opinion solicited in screening the alternatives.
- The third component is a collection

  $$C = \{C_1, \dots, C_n\}$$

  of criteria which are considered relevant in the choice of the objects to be further considered.

For each alternative each expert is required to provided his opinion. In particular for each alternative an expert is asked to evaluate how well that alternative satisfies each of the criteria in the set C. These evaluations of alternative satisfaction to criteria will be given in terms of elements from the following scale S:

Outstanding (OU)	S_7
Very High (VH)	S_6
High (H)	S_5
Medium (M)	S_4
Low	S_3
Very Low	S_2
None	S_1

The use of such a scale provides a natural ordering, $S_i > S_j$ if $i > j$ and the maximum and minimum of any two scores re defined by

$$\max(S_i, S_j) = S_i \text{ if } S_i \geq S_j,$$

$$\min(S_i, S_j) = S_j \text{ if } S_j \leq S_i$$

Thus for an alternative an expert provides a collection of n values

$$\{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$$

where $\mathcal{P}_j$ is the rating of the alternative on the j-th criteria by the expert. Each $\mathcal{P}_j$ is an element in the set of allowable scores S.

Assuming $n = 5$, a typical scoring for an alternative from one expert would be:

(medium, low, outstanding, very high, outstanding).

Independent of this evaluation of alternative satisfaction to criteria each expert must assign a measure of importance to each of the criteria. An expert uses the same scale, S, to provide the importance associated with the criteria.

The next step in the process is to find the overall valuation for a alternative by a given expert.

A crucial aspect here is the taking of the negation of the importances as

$$Neg(S_i) = S_{7-i+1}$$

For the scale that we are using, we see that the negation operation provides the following:

$$\begin{aligned}
Neg(OU) &= N \\
Neg(VH) &= VL \\
Neg(H) &= L \\
Neg(M) &= M \\
Neg(L) &= H \\
Neg(VL) &= VH \\
Neg(N) &= OU
\end{aligned}$$

Then the unit score of each alternative by each expert, denoted by $\mathcal{U}$, is calculated as follows

$$\mathcal{U} = \min_j \{Neg(I_j) \vee \mathcal{P}_j)\}$$

where I_j denotes the importance of the j-th critera.

We note that essentially is an *anding* of the criteria satisfactions modified by the importance of the criteria.

The formula can be seen as a measure of the degree to which an alternative satisfies the following proposition:

All important criteria are satisfied.

Example 1 *Consider some alternative with the following scores on five criteria*

Criteria:	C_1	C_2	C_3	C_4	C_5
Importance:	*VH*	*VH*	*M*	*L*	*VL*
Score:	*M*	*L*	*OU*	*VH*	*OU*

In this case we have

$$\mathcal{U} = \min\{Neg(VH) \vee M, Neg(VH) \vee L,$$

$$Neg(M) \vee OU, Neg(L) \vee VH, Neg(VL) \vee OU\} =$$

$$\min\{VL \vee M, VL \vee L, M \vee OU, H \vee VH, VH \vee OU\}$$

$$= \min\{M, L, OU, VH, OU\} = L$$

The essential reason for the low performance of this object is that it performed low on the second criteria which has a very high importance.

This formulation can be seen as a generalization of a weighted averaging. Linguistically, this formulation is saying that

> *If a criterion is important then an alternative should score well on it.*

As a result of the first stage, we have for each alternative a collection of evaluations

$$\{X_{i1}, X_{i2}, \ldots, X_{ir}\}$$

where X_{ik} is the unit evaluation of the i-th alternative by the k-th expert.

In the second stage the technique for combining the expert's evaluation to obtain an overall evaluation for each alternative is based upon the OWA operators.

The first step in this process is for the decision making body to provide an aggregation function, Q.

This function can be seen as a generalization of the idea of how many experts it feels need to agree on an alternative for it to be acceptable to pass the screening process.

In particular for each number i, where i runs from 1 to r, the decision making body must provide a value $Q(i)$ indicating how satisfied it would be in passing an alternative that i of the experts where satisfied with.

The values for $Q(i)$ should be drawn from the scale S described above.

It should be noted that Q should have certain characteristics to make it rational:

- As more experts agree the decision maker's satisfaction or confidence should increase

$$Q(i) \geq Q(j),\ i > j.$$

- If all the experts are satisfied then his satisfaction should be the highest possible

$$Q(r) = \text{Outstanding}.$$

A number for special forms for Q are worth noting:

- If the decision making body requires all experts to support an alternative then we get

$$\begin{aligned} Q(i) &= \text{ None} \quad \text{for } i > r \\ Q(r) &= \text{ Outstanding} \end{aligned}$$

- If the support of just one expert is enough to make a alternative worthy of consideration then

$$Q(i) = \text{Outstanding for all } i$$

- If at least m experts' support is needed for consideration then

$$\begin{aligned} Q(i) &= \text{ None} & i < j \\ Q(i) &= \text{ Outstanding} & i \geq m \end{aligned}$$

In order to define function Q, we introduce the operation

$$Int[a]$$

as returning the integer value that is closest to the number a.

In the following, we shall let q be the number of points on the scale and r be the number of experts participating.

This function which emulates the average is denoted as $Q_A(k)$ and is defined by

$$Q_A(k) = S_{b(k)}$$

where

$$b(k) = Int[1 + (k \times \frac{q-1}{r})]$$

for all $k = 0, 1, \ldots, r$.

We note that whatever the values of q and r it is always the case that

$$Q_A(0) = S_1, \quad Q_A(r) = S_q$$

As an example of this function if $r = 3$ and $q = 7$ then

$$b(k) = Int[1 + (k \times \frac{6}{3})] = Int[1 + 2k]$$

and

$$Q_A(0) = S_1, Q_A(1) = S_3, Q_A(2) = S_5, Q_A(3) = S_7.$$

If $r = 4$ and $q = 7$ then

$$b(k) = Int[1 + k \times 1.5]$$

and

$$Q_A(0) = S_1, \quad Q_A(1) = S_3, \quad Q_A(2) = S_4,$$
$$Q_A(3) = S_6, \quad Q_A(4) = S_7.$$

Having appropriately selected Q we are now in the position to use the OWA method for aggregating the expert opinions.

Assume we have r experts, each of which has a unit evaluation for the i-th projected denoted X_{ik}.

The first step in the OWA procedure is to order the X_{ik}'s in descending order, thus we shall denote B_j as the j-th highest score among the experts unit scores for the project.

To find the overall evaluation for the ith project, denoted X_i, we calculate

$$X_i = \max_j \{Q(j) \wedge B_j\}.$$

In order to appreciate the workings for this formulation we must realize that

- B_j can be seen as the worst of the j-th top scores.

- $Q(j) \wedge B_j$ can be seen as an indication of how important the decision maker feels that the support of at least j experts is.
- The term $Q(j) \wedge B_j$ can be seen as a weighting of an objects j best scores, B_j, and the decision maker requirement that j people support the project, $Q(j)$.
- The max operator plays a role akin to the summation in the usual numeric averaging procedure.

Example 2 *Assume we have four experts each providing a unit evaluation for project i obtained by the methodology discussed in the previous section.*

$$X_{i1} = M, X_{i2} = H, X_{i3} = H, X_{i4} = VH,$$

Reording these scores we get

$$B_1 = VH, B_2 = H, B_3 = H, B_4 = M$$

Furthermore, we shall assume that our decision making body chooses as its aggregation function the average like function Q_A. Then with $r = 4$ and scale cardinality $q = 7$, we obtain:

$$\begin{array}{lll} Q_A(1) = & L & (S_3) \\ Q_A(2) = & M & (S_4) \\ Q_A(3) = & VH & (S_6) \\ Q_A(4) = & OU & (S_7) \end{array}$$

We calculate the overall evaluation as

$$\begin{array}{ll} X_i = & \max\{L \wedge VH, M \wedge H, VH \wedge H, OU \wedge M\} \\ X_i = & \max\{L, M, H, M\} \\ X_i = & H \end{array}$$

Thus the overall evaluation of this alternative is high.

OWA operators: Further issues

In order to classify OWA operators in regard to their location between *and* and *or*, a measure of *orness*, associated with any vector W is introduce by Yager as follows

$$orness(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$$

Given a lingusitic quantifier Q if we generate the wights by

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n})$$

we can associate with this quantifier a degree of orness as

$$orness(Q) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\Big[Q(\frac{i}{n}) - Q(\frac{i-1}{n})\Big]$$

Algebraic manipulation of the formula leads to the form

$$orness(Q) = \frac{1}{n-1}\sum_{i=1}^{n} Q(\frac{i}{n})$$

Furthermore iw we let $b \to \infty$ then we can show that

$$orness(Q) = \int_0^1 Q(r)\,dr$$

The standard degree of orness associated with a RIM

Regular **I**ncreasing **M**onotone

linguistic quantifier is equal to the area under the quantifier.

This standard definition for the measure of orness of quantifier provides a simple useful method for obtaining this measure.

If we consider the quantifier

$$Q(r) = r^{\alpha},\ \alpha \geq 0,$$

then

$$orness(Q) = \int_0^1 r^{\alpha}\, dr = \frac{1}{\alpha + 1}$$

A number of interesting properties can be associated with this measure of orness.

If Q_1 and Q_2 are two quantifiers such that

$$Q_1(r) \geq Q_2(r)$$

for all r then

$$orness(Q_1) \geq orness(Q_2)$$

In addition since any regular quantifier is normal we see that

$$orness(Q) = 0 \leftrightarrow Q = Q_*$$

where q_* is the quantifier *for all*.

If we consider again the quantifier

$$Q(r) = r^{\alpha},\ \alpha \geq 0,$$

then there are three special cases of these family are worth noting

- For $\alpha = 1$ we get $Q(r) = r$. This called the *unitor* quantifier.
- For $\alpha \to \infty$ we get Q_*, the *universal* quantifier.
- For $\alpha \to 0$ we get Q^*, the *existential* quantifier.

A proportional type quantifier, such as *most*, can be represented as a fuzzy set Q of the segment $[0,1]$, where Q is the linguistic quantifier.

In this representation, the fuzzy set Q is defined such that for each $r \in [0,1]$, the membership grade, $Q(r)$, indicates the degree to which the proportion r satisfies the concept, linguistic quantifier, which Q is representing.

Assume $X = \{x_1, \ldots, x_n\}$ is some collection of objects and A is some concept expressed as a fuzzy set of X. A quantified proposition is a statement of the form

$$Q\ X\text{'s are } A$$

Examples of these kinds of statements

Most students are young

The truth value of these quantified propositions can be evaluated using the OWA aggregation operator. Once having obtained the wights from quantifier Q, we can obtain the truth τ, of the proposition

$$Q\ X\text{'s are } A$$

as

$$\tau = \sum_{i=1}^{n} w_i b_i$$

where b_i is the i-th largest element from the bag

$$< A(x_1), A(x_2) \ldots, A(x_n) > .$$

Let us have five students with ages $\{21, 22, 24, 22, 38\}$, and suppose that the degrees of *youngness* of students obtained from a fuzzy set for *young students*

$$A(u) = \begin{cases} 1 & \text{if } u \le 22 \\ 1 - (u-22)/8 & \text{if } 22 \le u \le 30 \\ 0 & \text{otherwise} \end{cases}$$

That is $A(21) = 1$, $A(22) = 1$, $A(24) = 0.75$, $A(22) = 1$ and $A(38) = 0$. If we order the bag

$$< 1, 1, 0.75, 1, 0 >$$

in monotone decreasing order then the b_i's look like

$$(1, 1, 1, 0.75, 0).$$

Let us define the quantifier *most* by the following membership function

$$Q(r) = \begin{cases} 1 & \text{if } r \ge 0.8 \\ 1 - (0.8-r)/0.6 & \text{if } 0.2 \le r \le 0.8 \\ 0 & \text{otherwise} \end{cases}$$

So, the OWA weights are

$$w_1 = Q(0.2) - Q(0) = 0, \ w_2 = Q(0.4) - Q(0.2) = 1/3,$$

$$w_3 = Q(0.6) - Q(0.4) = 1/3, \ w_4 = Q(0.8) - Q(0.6) = 1/3,$$

and $w_5 = Q(1) - Q(0.8) = 0$.

Then the degree of truth (which can be condsidered as the unit score of the bag, if the elements of the bag are criteria satisfactions) is

$$\tau = 0 \times 1 + 1/3 \times 1 + 1/3 \times 1 + 1/3 \times 0.75 + 0 \times 0 = \frac{11}{12}.$$