FD II: Aggregation operators

Triangular norms were introduced by Schweizer and Sklar

- B.Schweizer and A.Sklar, Associative functions and abstract semigroups, *Publ. Math. Debrecen*, 10(1963) 69-81.

to model the distances in probabilistic metric spaces. In fuzzy sets theory triangular norms are extensively used to model logical connective *and*.

Definition 1 *(Triangular norm.) A mapping*

$$T\colon [0,1] \times [0,1] \to [0,1]$$

is a triangular norm (t-norm for short) iff it is symmetric, associative, non-decreasing in each argument and $T(a,1) = a$, *for all* $a \in [0,1]$. *In other words, any t-norm* T *satisfies the properties:*

$T(x,y) = T(y,x)$ *(symmetricity)*

$T(x,T(y,z)) = T(T(x,y),z)$ *(associativity)*

$T(x,y) \leq T(x',y')$ *if* $x \leq x'$ *and* $y \leq y'$ *(monotonicity)*

$T(x,1) = x, \ \forall x \in [0,1]$ *(one identy)*

- minimum

$$MIN(a,b) = \min\{a,b\}$$

- Łukasiewicz

$$LAND(a,b) = \max\{a+b-1,0\}$$

• probabilistic

$$PAND(a, b) = ab$$

• weak

$$WEAK(a, b) = \begin{cases} \min\{a, b\} & \text{if } \max\{a, b\} = 1 \\ 0 & \text{otherwise} \end{cases}$$

• Hamacher

$$HAND_\gamma(a, b) = \frac{ab}{\gamma + (1 - \gamma)(a + b - ab)},\ \gamma \geq 0$$

• Dubois and Prade

$$DAND_\alpha(a, b) = \frac{ab}{\max\{a, b, \alpha\}},\ \alpha \in (0, 1)$$

All t-norms may be extended, through associativity, to $n > 2$ arguments. The t-norm MIN is automatically extended and

$$PAND(a_1, \ldots, a_n) = a_1 a_2 \cdots a_n$$

$$LAND(a_1, \ldots a_n) = \max\{\sum_{i=1}^{n} a_i - n + 1, 0\}$$

A t-norm T is called strict if T is strictly increasing in each argument.

Triangular conorms are extensively used to model logical connective *or*.

Definition 2 *(Triangular conorm.) A mapping*

$$S\colon [0, 1] \times [0, 1] \to [0, 1]$$

is a triangular co-norm (t-conorm for short) if it is symmetric, associative, non-decreasing in each argument and $S(a, 0) = a$*, for all* $a \in [0, 1]$*. In other words, any t-conorm* S *satisfies the properties:*

$$S(x,y) = S(y,x) \quad \textit{(symmetricity)}$$

$$S(x,S(y,z)) = S(S(x,y),z) \quad \textit{(associativity)}$$

$$S(x,y) \le S(x',y') \textit{ if } x \le x' \textit{ and } y \le y' \quad \textit{(monotonicity)}$$

$$S(x,0) = x, \ \forall x \in [0,1] \quad \textit{(zero identy)}$$

If T is a t-norm then the equality $S(a,b) := 1 - T(1-a, 1-b)$ defines a t-conorm and we say that S is derived from T.

- maximum $\quad MAX(a,b) = \max\{a,b\}$
- Łukasiewicz $\quad LOR(a,b) = \min\{a+b, 1\}$
- probabilistic $\quad POR(a,b) = a + b - ab$
- strong

$$STRONG(a,b) = \begin{cases} \max\{a,b\} & \text{if } \min\{a,b\} = 0 \\ 1 & \text{otherwise} \end{cases}$$

- Hamacher

$$HOR_\gamma(x,y) = \frac{a + b - (2-\gamma)ab}{1 - (1-\gamma)ab}, \ \gamma \ge 0$$

Lemma 1 *Let T be a t-norm. Then the following statement holds*

$$WEAK(x,y) \le T(x,y) \le \min\{x,y\}, \ \forall x,y \in [0,1].$$

Proof. From monotonicity, symmetricity and the extremal condition we get

$$T(x,y) \le T(x,1) \le x, \ T(x,y) = T(y,x) \le T(y,1) \le y.$$

This means that $T(x,y) \le \min\{x,y\}$.

Lemma 2 *Let S be a t-conorm. Then the following statement holds*

$$\max\{a, b\} \leq S(a, b) \leq STRONG(a, b), \ \forall a, b \in [0, 1]$$

Proof. From monotonicity, symmetricity and the extremal condition we get

$$S(x, y) \geq S(x, 0) \geq x, \ S(x, y) = S(y, x) \geq S(y, 0) \geq y$$

This means that $S(x, y) \geq \max\{x, y\}$.

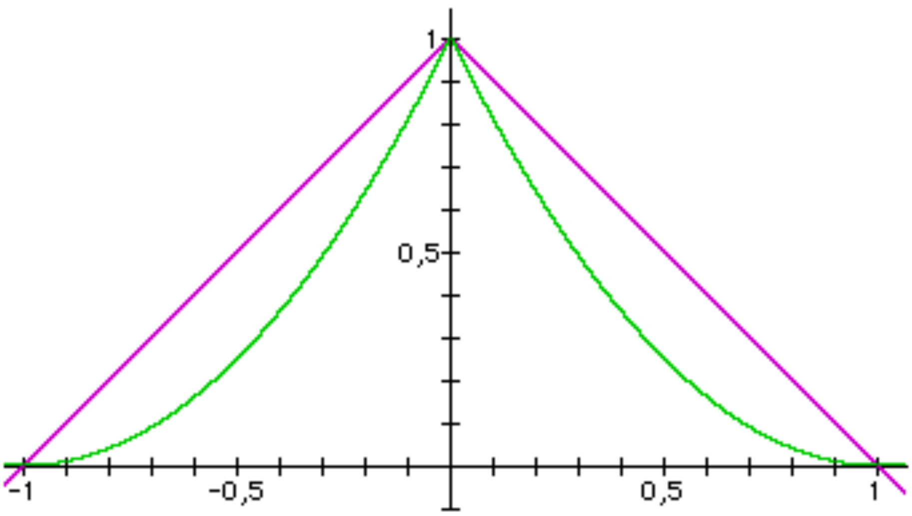
The operation *intersection* can be defined by the help of triangular norms.

Definition 3 *(t-norm-based intersection) Let T be a t-norm. The T-intersection of A and B is defined as*

$$(A \cap B)(t) = T(A(t), B(t)), \ \forall t \in X.$$

Example 1 *Let $T(x, y) = xy$ be the product t-norm. Then we have*

$$(A \cap B)(t) = A(t)B(t) \ \forall t \in X.$$



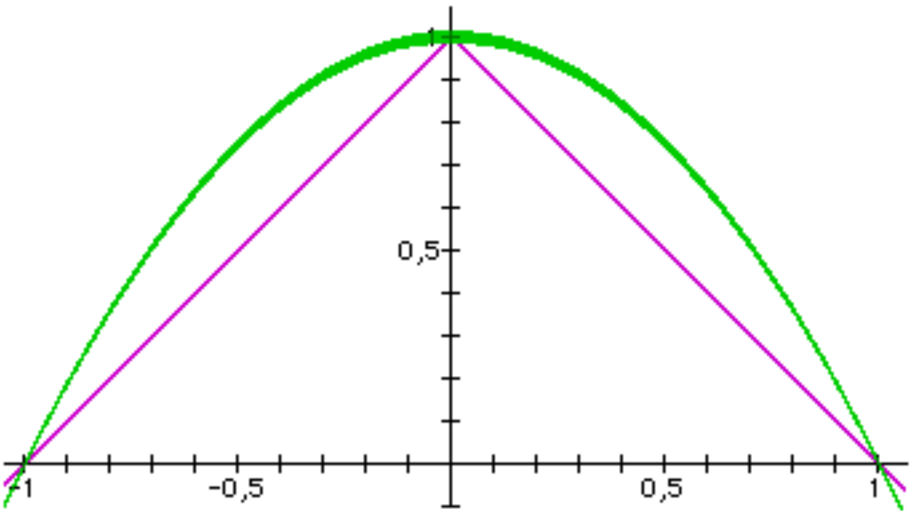
$$A(t) = B(t) = 1 - |t|.$$

Definition 4 *(t-conorm-based union) Let S be a t-conorm. The S-union of A and B is defined as*

$$(A \cup B)(t) = S(A(t), B(t)), \ \forall t \in X.$$

Example 2 *Let $S(x, y) = x + y - xy$ be the probabilistic sum. Then we have*

$$(A \cup B)(t) = A(t) + B(t) - A(t)B(t), \ \forall t \in X.$$



$A(t) = B(t) = 1 - |t|$.

Averaging operators

In a decision process the idea of *trade-offs* corresponds to viewing the global evaluation of an action as lying between the *worst* and the *best* local ratings.
This occurs in the presence of conflicting goals, when a compensation between the corresponding compabilities is allowed.
Averaging operators realize trade-offs between objectives, by allowing a positive compensation between ratings.

Definition 5 *(averaging operator) An averaging (or mean) operator M is a function*

$$M\colon [0,1] \times [0,1] \to [0,1]$$

satisfying the following properties

- $M(x,x) = x, \ \forall x \in [0,1],$ *(idempotency)*
- $M(x,y) = M(y,x), \ \forall x,y \in [0,1],$ *(commutativity)*
- $M(0,0) = 0, \ M(1,1) = 1,$ *(extremal conditions)*
- $M(x,y) \le M(x',y')$ *if* $x \le x'$ *and* $y \le y'$ *(monotonicity)*
- M *is continuous*

Lemma 3 *If M is an averaging operator then*

$$\min\{x,y\} \le M(x,y) \le \max\{x,y\}, \ \forall x,y \in [0,1]$$

Proof. From idempotency and monotonicity of M it follows that

$$\min\{x,y\} = M(\min\{x,y\}, \min\{x,y\}) \le M(x,y)$$

and

$$M(x,y) \le M(\max\{x,y\}, \max\{x,y\}) = \max\{x,y\}.$$

Which ends the proof.

An important family of averaging operators is formed by quasi-arithmetic means

$$M(a_1, \dots, a_n) = f^{-1}(\frac{1}{n} \sum_{i=1}^{n} f(a_i))$$

This family has been characterized by Kolmogorov as being the class of all decomposable continuous averaging operators.

Example 3 *For example, the quasi-arithmetic mean of a_1 and a_2 is defined by*

$$M(a_1, a_2) = f^{-1}\Big[\frac{f(a_1) + f(a_2)}{2}\Big].$$

The most often used mean operators.

harmonic mean

$$\frac{2xy}{x+y}$$

geometric mean

$$\sqrt{xy}$$

arithmetic mean

$$\frac{x+y}{2}$$

dual of geometric mean

$$1 - \sqrt{(1-x)(1-y)}$$

dual of harmonic mean

$$\frac{x+y-2xy}{2-x-y}$$

median

$$\mathrm{med}\,(x, y, \alpha) = \begin{cases} y & \text{if } x \le y \le \alpha \\ \alpha & \text{if } x \le \alpha \le y \\ x & \text{if } \alpha \le x \le y \end{cases}$$

generalized p-mean

$$\Big[\frac{x^p + y^p}{2}\Big]^{1/p},\ p \ge 1$$

The process of information aggregation appears in many applications related to the development of intelligent systems. One sees aggregation in neural networks, fuzzy logic controllers, vision systems, expert systems and multi-criteria decision aids. In

- R.R.Yager, Ordered weighted averaging aggregation operators in multi-criteria decision making, *IEEE Trans. on Systems, Man and Cybernetics*, 18(1988) 183-190.

Yager introduced a new aggregation technique based on the ordered weighted averaging (OWA) operators.

Definition 6 *An OWA operator of dimension n is a mapping*

$$F\colon \mathbf{R}^n \to \mathbf{R},$$

that has an associated n vector

$$W = (w_1, w_2, \dots, w_n)^T$$

such as $w_i \in [0,1]$, $1 \leq i \leq n$,

$$\sum_{i=1}^{n} w_i = 1.$$

Furthermore

$$F(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j b_j$$

where b_j is the j-th largest element of the bag

$$< a_1, \dots, a_n > .$$

Example 4 *Assume*

$$W = (0.4, 0.3, 0.2, 0.1)^T$$

then

$$F(0.7, 1, 0.2, 0.6) = 0.4 \times 1 + 0.3 \times 0.7 + 0.2 \times 0.6 + 0.1 \times 0.2 = 0.75.$$

A fundamental aspect of this operator is the re-ordering step, in particular an aggregate a_i is not associated with a particular weight w_i but rather a weight is associated with a particular ordered position of aggregate.

When we view the OWA weights as a column vector we shall find it convenient to refer to the weights with the low indices as weights at the top and those with the higher indices with weights at the bottom.

It is noted that different OWA operators are distinguished by their weighting function.

Yager pointed out three important special cases of OWA aggregations:

- F^*: In this case $W = W^* = (1, 0 \ldots, 0)^T$ and
$$F^*(a_1, \ldots, a_n) = \max\{a_1, \ldots, a_n\},$$
- F_*: In this case $W = W_* = (0, 0 \ldots, 1)^T$ and
$$F_*(a_1, \ldots, a_n) = \min\{a_1, \ldots, a_n\},$$
- F_A: In this case $W = W_A = (1/n, \ldots, 1/n)^T$ and
$$F_A(a_1, \ldots, a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i,$$

A number of important properties can be associated with the OWA operators. We shall now discuss some of these.

For any OWA operator F
$$F_*(a_1, \ldots, a_n) \le F(a_1, \ldots, a_n) \le F^*(a_1, \ldots, a_n).$$

Thus the upper an lower star OWA operator are its boundaries. From the above it becomes clear that for any F
$$\max\{a_1, \ldots, a_n\} \le F(a_1, \ldots, a_n) \le \max\{a_1, \ldots, a_n\}.$$

The OWA operator can be seen to be *commutative*.

Let $\{a_1, \dots, a_n\}$ be a bag of aggregates and let $\{d_1, \dots, d_n\}$ be any *permutation* of the a_i.

Then for any OWA operator

$$F(a_1, \dots, a_n) = F(d_1, \dots, d_n).$$

A second characteristic associated with these operators is *monotonicity*.

Assume a_i and c_i are a collection of aggregates, $i = 1, \dots, n$ such that for each i, $a_i \geq c_i$. Then

$$F(a_1, \dots, a_n) \geq F(c_1, c_2, \dots, c_n)$$

where F is some fixed weight OWA operator.

Another characteristic associated with these operators is *idempotency*.

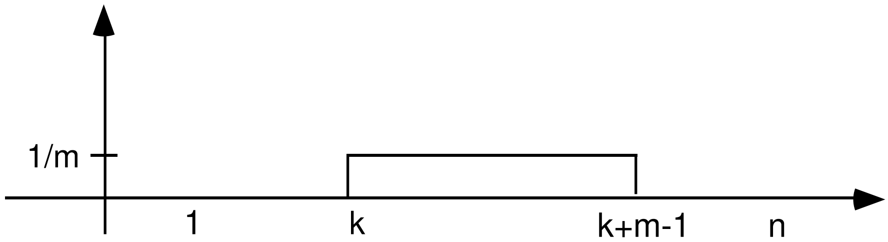
If $a_i = a$ for all i then for any OWA operator

$$F(a_1, \dots, a_n) = F(a, \dots, a) = a.$$

From the above we can see the OWA operators have the basic properties associated with an *averaging operator*.

Example 5 *A window type OWA operator takes the average of the m arguments about the center. For this class of operators we have*

$$w_i = \begin{cases} 0 & \mathit{if}\ i < k \\ 1/m & \mathit{if}\ k \leq i < k+m \\ 0 & \mathit{if}\ i \geq k+m \end{cases}$$



Window type OWA operator.

In order to classify OWA operators in regard to their location between *and* and *or*, a measure of *orness*, associated with any vector W is introduce by Yager as follows

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i$$

$$orness(W) = \frac{1}{n-1} \times \Big[(n-1) w_1 + \cdots + w_{n-1} \Big]$$

$$orness(W) = w_1 + \frac{n-2}{n-1} \times w_2 + \cdots + \frac{1}{n-1} \times w_{n-1}$$

It is easy to see that for any W the $orness(W)$ is always in the unit interval. Furthermore, note that the nearer W is to an *or*, the closer its measure is to one; while the nearer it is to an *and*, the closer is to zero.

Lemma 4 *Let us consider the the vectors*

$$W^* = (1, 0 \ldots, 0)^T, \quad W_* = (0, 0 \ldots, 1)^T,$$

$$W_A = (1/n, \ldots, 1/n)^T.$$

Then

$$orness(W^*) = 1, \quad orness(W_*) = 0$$

$$orness(W_A) = 0.5$$

Proof

$$orness(W^*) = \frac{1}{n-1}((n-1)w_1 + \cdots + w_{n-1}) =$$

$$\frac{1}{n-1}((n-1) + \cdots + 0) = 1.$$

$$orness(W_*) = \frac{1}{n-1}((n-1)w_1 + \cdots + w_{n-1}) =$$

$$\frac{1}{n-1}(0 + \cdots + 0) = 0.$$

$$orness(W_A) = \frac{1}{n-1}\Big[\frac{n-1}{n} + \cdots + \frac{1}{n}\Big] =$$

$$\frac{n(n-1)}{2n(n-1)} = 0.5.$$

A measure of *andness* is defined as

$$andness(W) = 1 - orness(W)$$

Generally, an OWA opeartor with much of nonzero weights near the top will be an *orlike* operator,

$$orness(W) \geq 0.5$$

and when much of the weights are nonzero near the bottom, the OWA operator will be *andlike* operator

$$andness(W) \geq 0.5.$$

Example 6 *Let* $W = (0.8, 0.2, 0.0)^T$. *Then*

$$orness(W) = \frac{1}{3}(2 \times 0.8 + 0.2) = 0.6$$

and

$$andness(W) = 1 - orness(W) = 1 - 0.6 = 0.4.$$

This means that the OWA operator, defined by

$$F(a_1, a_2, a_3) = 0.8b_1 + 0.2b_2 + 0.0b_3 = 0.8b_1 + 0.2b_2$$

where b_j *is the* j*-th largest element of the bag*

$$< a_1, a_2, a_3 >,$$

is an orlike *aggregation.*

Suppose we have n appliccants for a Ph.D. program. Each application is evaluated by experts, who provides ratings on each of the criteria from the set

- 3 (high)
- 2 (medium)
- 1 (low)

Compensative connectives have the property that a higher degree of satisfaction of one of the criteria can compensate for a lower degree of satisfaction of another criteria.

Oring the criteria means full compensation and *Anding* the criteria means no compensation.

We illustrate the effect of compensation rate on the overall rating:

Let us have the following ratings

$$(3, 2, 1)$$

If $w = (w_1, w_2, w_3)$ is an OWA weight then

$$orness(w_1, w_2, w_3) = w_1 + \frac{1}{2}w_2.$$

Min operator: the overalling rating is

$$\min\{3, 2, 1\} = 1$$

in this case there is no compensation, because

$$orness(0, 0, 1) = 0.$$

Max operator: the overalling rating is

$$\max\{3, 2, 1\} = 3$$

in this case there is full compensation, because

$$orness(1, 0, 0) = 1.$$

Mean operator: the overalling rating is

$$\frac{1}{3}(3 + 2 + 1) = 2$$

in this case the measure of compensation is 0.5, because

$$orness(1/3, 1/3, 1/3) = 1/3 + 1/2 \times 1/3 = 1/2.$$

An andlike operator: the overalling rating is

$$0.2 \times 3 + 0.1 \times 2 + 0.7 \times 1 = 1.5$$

in this case the measure of compensation is 0.3, because

$$orness(0.2, 0.1, 0.7) = 0.2 + 1/2 \times 0.2 = 0.3$$

An orlike operator: the overalling rating is

$$0.6 \times 3 + 0.3 \times 2 + 0.1 \times 1 = 2.5$$

in this case the measure of compensation is 0.75, because

$$orness(0.6, 0.3, 0.1) = 0.6 + 1/2 \times 0.3 = 0.75$$

The following theorem shows that as we move weight up the vector we increase the orness, while moving weight down causes us to decrease $orness(W)$.

Theorem 1 *Assume W and W' are two n-dimensional OWA vectors such that*

$$W = (w_1, \dots, w_n)^T,$$

$$W' = (w_1, \dots, w_j + \epsilon, \dots, w_k - \epsilon, \dots, w_n)^T$$

where $\epsilon > 0$, $j < k$. Then $orness(W') > orness(W)$.

Proof. From the definition of the measure of *orness* we get

$$orness(W') = \frac{1}{n-1} \sum_i (n-i) w'_i =$$

$$\frac{1}{n-1} \sum_i (n-1) w_i + (n-j)\epsilon - (n-k)\epsilon,$$

$$orness(W') = orness(W) + \frac{1}{n-1} \epsilon (k-j).$$

Since $k > j$, $orness(W') > orness(W)$.

Yager defined the measure of dispersion (or entropy) of an OWA vector by

$$disp(W) = -\sum_i w_i \ln w_i.$$

We can see when using the OWA operator as an averaging operator $Disp(W)$ measures the degree to which we use all the aggregates equally.

If F is an OWA aggregation with weights w_i the *dual* of F denoted $\hat{F}$, is an OWA aggregation of the same dimention where with weights $\hat{w}_i$

$$\hat{w}_i = w_{n-i+1}.$$

We can easily see that if F and $\hat{F}$ are duals then

$$disp(\hat{F}) = disp(F)$$

$$orness(\hat{F}) = 1 - orness(F) = andness(F)$$

Thus is F is orlike its dual is andlike.

Example 7 *Let* $W = (0.3, 0.2, 0.1, 0.4)^T$. *Then*

$$\hat{W} = (0.4, 0.1, 0.2, 0.3)^T.$$

and

$$orness(F) = 1/3(3 \times 0.3 + 2 \times 0.2 + 0.1) \approx 0.466$$
$$orness(\hat{F}) = 1/3(3 \times 0.4 + 2 \times 0.1 + 0.2) \approx 0.533$$

An important application of the OWA operators is in the area of quantifier guided aggregations.

Assume

$$\{A_1, \dots, A_n\}$$

is a collection of criteria.

Let x be an object such that for any criterion A_i, $A_i(x) \in [0, 1]$ indicates the degree to which this criterion is satisfied by x.

If we want to find out the degree to which x satisfies *"all* the criteria" denoting this by $D(x)$, we get following Bellman and Zadeh.

$$D(x) = \min\{A_1(x), \dots, A_n(x)\}$$

In this case we are essentially requiring x to satisfy

$$A_1 \text{ and } A_2 \text{ and } \dots \text{ and } A_n.$$

If we desire to find out the degree to which x satisfies *"at least one* of the criteria", denoting this $E(x)$, we get

$$E(x) = \max\{A_1(x), \dots, A_n(x)\}$$

In this case we are requiring x to satisfy

$$A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n.$$

In many applications rather than desiring that a solution satisfies one of these extreme situations, "all" or "at least one", we may require that x satisfies *most* or *at least half* of the criteria.

Drawing upon Zadeh's concept of linguistic quantifiers we can accomplish these kinds of quantifier guided aggregations.

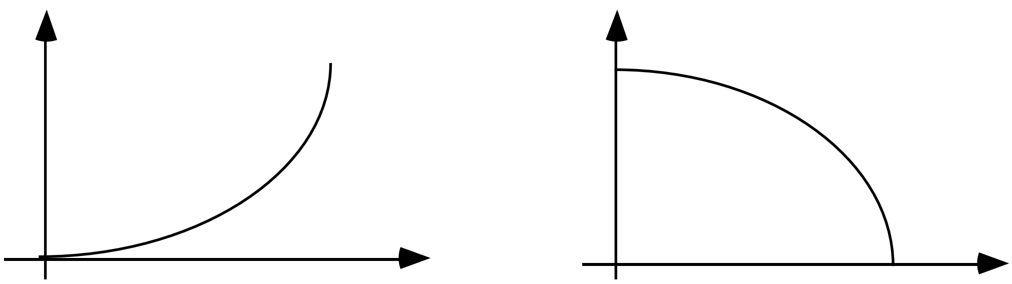
Definition 7 *A quantifier Q is called*

- regular monotonically non-decreasing *if*

$$Q(0) = 0, \quad Q(1) = 1, \quad \textit{if } r_1 > r_2 \textit{ then } Q(r_1) \geq Q(r_2).$$

- regular monotonically non-increasing *if*

$$Q(0) = 1, \quad Q(1) = 0, \quad \textit{if } r_1 < r_2 \textit{ then } Q(r_1) \geq Q(r_2).$$



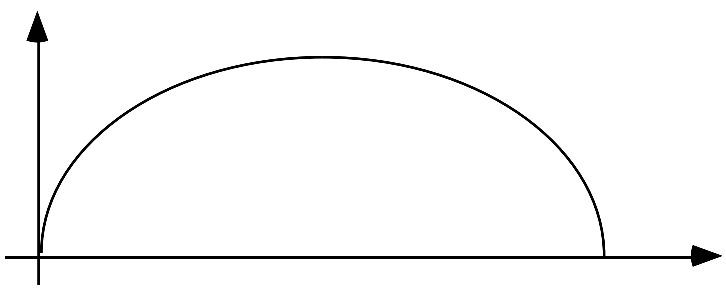
Monoton linguistic quantifiers.

- regular unimodal *if*

$$Q(0) = Q(1) = 0, \quad Q(r) = 1 \textit{ for } a \leq r \leq b,$$

$$r_2 \leq r_1 \leq a \textit{ then } Q(r_1) \geq Q(r_2),$$

$$r_2 \geq r_1 \geq b \textit{ then } Q(r_2) \leq Q(r_1).$$



Unimodal linguistic quantifier.

With $a_i = A_i(x)$ the overall valuation of x is

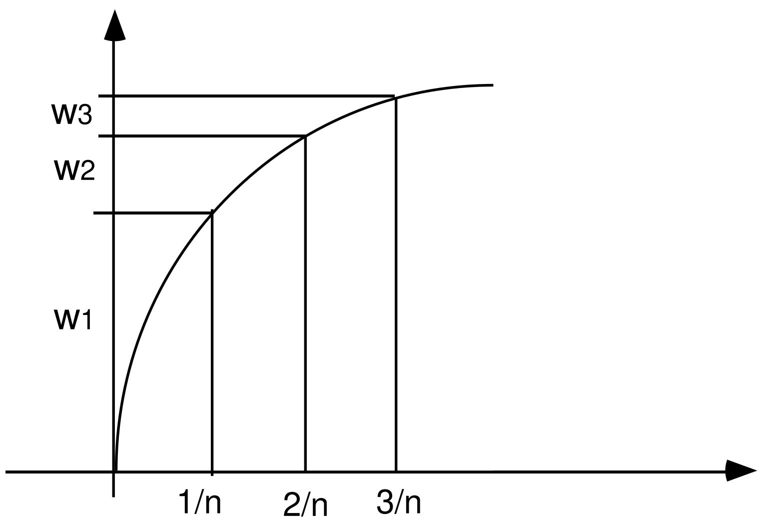
$$F_Q(a_1, \dots, a_n)$$

where F_Q is an OWA operator.

The weights associated with this quantified guided aggregation are obtained as follows

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}),\ i = 1, \dots, n.$$

The next figure graphically shows the operation involved in determining the OWA weights directly from the quantifier guiding the aggregation.



Determining weights from a quantifier.

Theorem 2 *If we construct w_i via the method above we always get*

$$\sum w_i = 1$$
$$w_i \in [0, 1]$$

for any function

$$Q\colon [0, 1] \to [0, 1]$$

satisfying the conditions of a regular nondecreasing quantifier.

Proof. We first see that from the non-decreasing property

$$Q(i/n) \geq Q(i-1/n)$$

hence $w_i \geq 0$ and since $Q(r) \leq 1$ then $w_i \leq 1$.

Furthermore we see

$$\sum_i w_i = \sum_i (Q(\frac{i}{n}) - Q(\frac{i}{n-1})) - Q(\frac{n}{n}) - Q(\frac{0}{n}) = 1 - 0 = 1.$$

we call any function satisfying the conditions of a regular non-decreasing quantifier an *acceptable OWA weight generating function.*

Let us look at the weights generated from some basic types of quantifiers. The quantifier, *for all* Q_*, is defined such that

$$Q_*(r) = \begin{cases} 0 & \text{for } r < 1, \\ 1 & \text{for } r = 1. \end{cases}$$

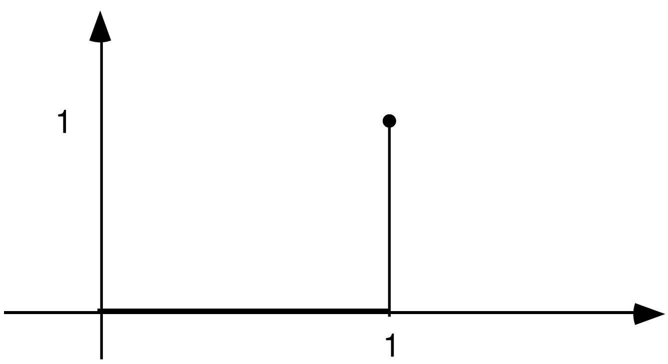
Using our method for generating weights

$$w_i = Q_*(\frac{i}{n}) - Q_*(\frac{i-1}{n})$$

we get

$$w_i = \begin{cases} 0 & \text{for } i < n, \\ 1 & \text{for } i = n. \end{cases}$$

This is exactly what we previously denoted as W_*.



The quantifier *all*.

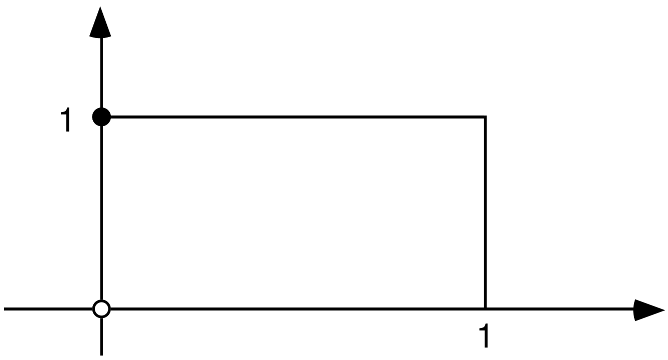
For the quantifier *there exists* we have

$$Q^*(r) = \begin{cases} 0 & \text{for } r = 0, \\ 1 & \text{for } r > 0. \end{cases}$$

In this case we get

$$w_1 = 1, \quad w_i = 0, \text{ for } i \neq 1.$$

This is exactly what we denoted as W^*.

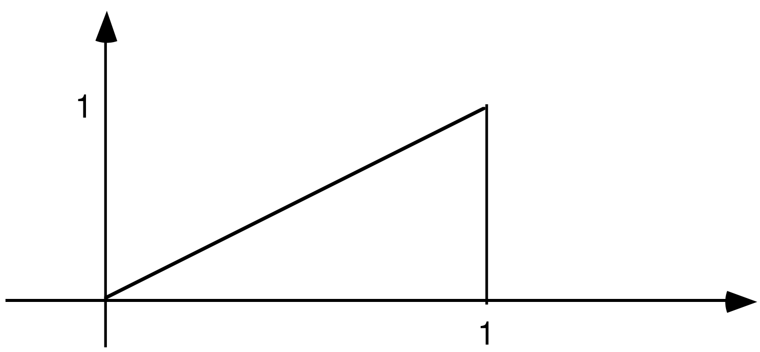


The quantifier *there exists*.

Consider next the quantifier defined by

$$Q(r) = r.$$

This is *an identity* or *linear type* quantifier.



The *identity* quantifier.

In this case we get

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}) = \frac{i}{n} - \frac{i-1}{n} = \frac{1}{n}.$$

This gives us the pure averaging OWA aggregation operator.

Recapitulating using the approach suggested by Yager if we desire to calculate

$$F_Q(a_1, \ldots, a_n)$$

for Q being a regular non-decreasing quantifier we proceed as follows:

- (1) Calculate
$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}),$$
- (2) Calculate
$$F_Q(a_i, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i$$

where b_i is the i-th largest of the a_j.

Exercise 1 *Let* $W = (0.4, 0.2, 0.1, 0.1, 0.2)^T$*. Calculate* $disp(W)$*.*

Exercise 2 *Let* $W = (0.3, 0.3, 0.1, 0.1, 0.2)^T$*. Calculate* $orness(F)$*, where the OWA operator* F *is derived from* W*.*

Exercise 3 *Prove that* $0 \leq disp(W) \leq \ln(n)$ *for any* n*-dimensional weight vector* W*.*

Exercise 4 *Let* $Q(x) = x^2$ *be a linguistic quintifier. Assume the weights of an OWA operator* F *are derived from* Q*. Calculate the value* $F(a_1, a_2, a_3, a_4)$ *for* $a_1 = a_2 = 0.6$*,* $a_3 = 0.4$ *and* $a_4 = 0.2$*. What is the orness measure of* F*?*

Exercise 5 *Let* $Q(x) = \sqrt{x}$ *be a linguistic quintifier. Assume the weights of an OWA operator* F *are derived from* Q*. Calculate the value* $F(a_1, a_2, a_3, a_4)$ *for* $a_1 = a_2 = 0.6$*,* $a_3 = 0.4$ *and* $a_4 = 0.2$*. What is the orness measure of* F*?*