Chapter 10
RC Circuits

Objectives

• Describe the relationship between current and voltage in an RC circuit
• Determine impedance and phase angle in a series RC circuit
• Analyze a series RC circuit
• Determine the impedance and phase angle in a parallel RC circuit
Objectives

- Analyze a parallel RC circuit
- Analyze series-parallel RC circuits
- Determine power in RC circuits

Sinusoidal Response of RC Circuits

- When a circuit is purely resistive, the phase angle between applied voltage and total current is zero
- When a circuit is purely capacitive, the phase angle between applied voltage and total current is 90°
- When there is a combination of both resistance and capacitance in a circuit, the phase angle between the applied voltage and total current is somewhere between 0° and 90°, depending on relative values of resistance and capacitance
Impedance and Phase Angle of Series RC Circuits

- In the series RC circuit, the total impedance is the phasor sum of $R$ and $jX_C$
- Impedance magnitude: $Z = \sqrt{R^2 + X_C^2}$
- Phase angle: $\theta = \tan^{-1}(X_C/R)$

Analysis of Series RC Circuits

- The application of Ohm’s law to series RC circuits involves the use of the quantities $Z$, $V$, and $I$ as:

$$V = IZ$$
$$I = V/Z$$
$$Z = V/I$$
Relationships of I and V in a Series RC Circuit

- In a series circuit, the current is the same through both the resistor and the capacitor.
- The resistor voltage is in phase with the current, and the capacitor voltage lags the current by 90°.

KVL in a Series RC Circuit

- From KVL, the sum of the voltage drops must equal the applied voltage ($V_S$).
- Since $V_R$ and $V_C$ are 90° out of phase with each other, they must be added as phasor quantities.
- Magnitude of source voltage:
  $$V_S = \sqrt{V^2_R + V^2_C}$$
- Phase angle between resistor and source voltages:
  $$\theta = \tan^{-1}(V_C/V_R)$$
Variation of Impedance and Phase Angle with Frequency

- For a series RC circuit; as frequency increases:
  - $X_C$ decreases
  - $Z$ decreases
  - $\theta$ decreases
  - $R$ remains constant

Impedance and Phase Angle of Parallel RC Circuits

- Total impedance:
  $$Z = \frac{RX_C}{\sqrt{R^2 + X_C^2}}$$
- Phase angle:
  $$\theta = \tan^{-1}(R/X_C)$$
Conductance, Susceptance and Admittance

- Conductance is the reciprocal of resistance:
  \[ G = \frac{1}{R} \]
- Capacitive susceptance is the reciprocal of capacitive reactance:
  \[ B_C = \frac{1}{X_C} \]
- Admittance is the reciprocal of impedance:
  \[ Y = \frac{1}{Z} \]

Ohm’s Law

- Application of Ohm’s Law to parallel RC circuits using impedance can be rewritten for admittance \((Y = 1/Z)\):
  \[
  \begin{align*}
  V &= \frac{I}{Y} \\
  I &= VY \\
  Y &= \frac{I}{V}
  \end{align*}
  \]
Relationships of the Currents and Voltages in a Parallel RC Circuit

- The applied voltage, \( V_S \), appears across both the resistive and the capacitive branches.
- Total current \( I_{\text{tot}} \) divides at the junction into the two branch currents, \( I_R \) and \( I_C \).

Kirchhoff’s Current Law

- Current through the resistor is in phase with the voltage.
- Current through the capacitor leads the voltage, and thus the resistive current by 90°.
- Total current is the phasor sum of the two branch currents.
- Magnitude of total current is:
  \[
  I_{\text{tot}} = \sqrt{I_R^2 + I_C^2}
  \]
- Phase angle: \( \theta = \tan^{-1}(I_C/I_R) \)
Conversion from Parallel to Series Form

- For every parallel RC circuit there is an equivalent series RC circuit for any given frequency
- Equivalent resistance and capacitive reactance are indicated on the impedance triangle

Series-Parallel RC Circuits

- An approach to analyzing circuits with combinations of both series and parallel R and C elements is to:
  - Calculate the magnitudes of capacitive reactances ($X_C$)
  - Find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance
Power in RC Circuits

- When there is both resistance and capacitance, some of the energy is alternately stored and returned by the capacitance and some is dissipated by the resistance
- The amount of energy converted to heat is determined by the relative values of the resistance and the capacitive reactance

Power Triangle for RC Circuits

- The Power can be written as:
  \[ P_{\text{true}} = V_s I_{\text{total}} \cos \theta \]

  where:  \( \theta = 0^\circ \) for a purely resistive circuit
           since \( \cos(0^\circ) = 1 \), \( P_{\text{true}} = V_s I_{\text{total}} \)

  \( \theta = 90^\circ \) for a purely capacitive circuit
           since \( \cos(90^\circ) = 0 \), \( P_{\text{true}} = \text{zero} \)
Power Factor

• The term \( \cos \theta \), in the previous slide, is called the power factor:

\[ \text{PF} = \cos \theta \]

• The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit
• In an RC circuit, the power factor is referred to as a leading power factor because the current leads the voltage

Significance of Apparent Power

• Apparent power is the power that appears to be transferred between the source and the load
• Apparent power consists of two components; a true power component, that does the work, and a reactive power component, that is simply power shuttled back and forth between source and load
• Apparent power is expressed in volt-amperes (VA)
RC Lag Network

- The RC lag network is a phase shift circuit in which the output voltage lags the input voltage

\[ V_{\text{out}} = \left( \frac{X_C}{\sqrt{R^2 + X_C^2}} \right) V_{\text{in}} \]

(a) A basic RC lag network

(b) Phase voltage diagram showing the phase lag between \( V_{\text{in}} \) and \( V_{\text{out}} \)

RC Lead Network

- The RC lead network is a phase shift circuit in which the output voltage leads the input voltage

\[ V_{\text{out}} = \left( \frac{R}{\sqrt{R^2 + X_C^2}} \right) V_{\text{in}} \]

(a) A basic RC lead network

(b) Phase voltage diagram showing the phase lead between \( V_{\text{in}} \) and \( V_{\text{out}} \)
Frequency Selectivity of RC Circuits

• Frequency-selective circuits permit signals of certain frequencies to pass from the input to the output, while blocking all others.
• A **low-pass circuit** is realized by taking the output across the capacitor, just as in a lag network.
• A **high-pass circuit** is implemented by taking the output across the resistor, as in a lead network.

Frequency Selectivity of RC Circuits

• The frequency at which the capacitive reactance equals the resistance in a low-pass or high-pass RC circuit is called the **cutoff frequency**: 
  \[ f_c = \frac{1}{2\pi RC} \]
Summary

• When a sinusoidal voltage is applied to an RC circuit, the current and all the voltage drops are also sine waves
• Total current in an RC circuit always leads the source voltage
• The resistor voltage is always in phase with the current
• The capacitor voltage always lags the current by $90^\circ$

Summary

• In an RC circuit, the impedance is determined by both the resistance and the capacitive reactance combined
• Impedance is expressed in units of ohms
• The circuit phase angle is the angle between the total current and the source voltage
• The impedance of a series RC circuit varies inversely with frequency
Summary

• The phase angle ($\theta$) of a series RC circuit varies
  inversely with frequency
• For each parallel RC circuit, there is an equivalent
  series circuit for any given frequency
• The impedance of a circuit can be determined by
  measuring the applied voltage and the total current
  and then applying Ohm’s law

Summary

• In an RC circuit, part of the power is resistive and
  part is reactive
• The phasor combination of resistive power and
  reactive power is called apparent power
• Apparent power is expressed in volt-amperes (VA)
• The power factor indicates how much of the
  apparent power is true power
Summary

- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit
- In a lag network, the output voltage lags the input voltage in phase
- In a lead network, the output voltage leads the input voltage
- A filter passes certain frequencies and rejects others