# Solution to Problem 3.3a: Order by asymptotic growth rates *

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First we simplify some of them, and classify them into exponential, polynomial, and poly-log functions.

**Class 1: Exponential (or higher than polynomial)**

$f_5 = n!$

$f_6 = (\lg n)! = \Theta(n^{\lg \lg n})$ since $\lg f_6 = \Theta(\lg n \lg \lg n)$ by Stirling's approximation.

$f_7 = (\frac{3}{2})^n$

$f_{11} = 2^{2^n}$

$f_{15} = n \cdot 2^n$

*in P.58 of the textbook "Introduction to Algorithms" by Cormen

$f_{16} = n^{\lg\lg n} = \Theta(f_6)$

$f_{20} = (\lg n)^{\lg n} = n^{\lg\lg n} = \Theta(f_6)$ since $\lg f_{20} = \lg n \lg\lg n$.

$f_{21} = e^n$

$f_{23} = (n+1)!$

$f_{28} = 2^n$

$f_{30} = 2^{2^{n+1}}$

**Class 2: Polynomial**

$f_3 = (\sqrt{2})^{\lg n} = (2^{\lg n})^{1/2} = \sqrt{n}$

$f_4 = n^2$

$f_8 = n^3$

$f_{10} = \lg(n!) = \Theta(n \lg n)$ by Stirling's approx.

$f_{19} = 2^{\lg n} = n$

$f_{22} = 4^{\lg n} = (2^{\lg n})^2 = n^2$

$f_{26} = 2^{\sqrt{2\lg n}}$ since $\lg f_{26} = \sqrt{2\lg n} = \Theta(\lg^{1/2} n)$, and then $f_{26} < n$

$f_{27} = n$

$f_{29} = n \lg n$

**Class 3: Poly-logarithm or smaller**

$f_1 = \lg(\lg^* n)$

$f_2 = 2^{\lg^* n}$

$f_9 = \lg^2 n$

$f_{12} = n^{1/\lg n} = 2$ since $\lg f_{12} = 1$

$f_{13} = \ln \ln n$

$f_{14} = \lg^* n$

$f_{17} = \ln n$

$f_{18} = 1$

$f_{24} = \sqrt{\lg n}$

$f_{25} = \lg^*(\lg n)$

**For Class 1:**

First

$$f_{21} = e^n > f_{15} = n \cdot 2^n > f_{28} = 2^n > f_7 = (\frac{3}{2})^n$$

Note Exponential functions with different bases are of different orders.

$$f_6 = \Theta(n^{\lg \lg n}) = f_{16} = f_{20}$$

$$f_{23} = (n+1)! = (n+1) \times f_5 > f_5$$

$$f_{30} = 2^{2^{n+1}} > f_{11} = 2^{2^n}$$

Since $\lg f_{30} = 2 \lg f_{11},\ f_{30} = (f_{11})^2$.

$\lg f_7 = cn > \lg f_6 = \lg n \lg \lg n$.

$\lg f_{11} = 2^n > \lg f_{23}$, $\lg f_5 = n \lg n > \lg f_{21} = cn$.

Finally, we have

$$f_{30} > f_{11} > f_{23} > f_5 > f_{21} > f_{15} > f_{28} > f_7 > f_6 = f_{16} = f_{20}$$

**For Class 2:**

$$f_8 = n^3 > f_4 = n^2 = f_{22} = n^2 > f_{10} = \Theta(n \lg n) = f_{29} = n \lg n$$
$$> \quad f_{27} = n = f_{19} > f_3 = \sqrt{n} > f_{26}$$

because $\lg f_{26} = \sqrt{2 \lg n} < \lg f_3 = (1/2) \lg n$.

**For Class 3:**

$$f_9 = \lg^2 n > f_{17} = \ln n > f_{24} = \sqrt{\lg n} > f_{13} = \ln \ln n > f_{12} = 2 = \Theta(1) = f_{18}$$

$$f_2 = 2^{\lg^* n} > f_{14} = \lg^* n > f_{25} = \lg^*(\lg n) > f_1 = \lg(\lg^* n)$$

To see the relation of the last two, suppose that $\lg^* n = k$. Then, $f_{25} = k - 1$ and $f_1 = \lg k$.

Note $\lg^* n$ is almost constant (but not). Where should the second sub-class be in the first row?

$$\lg f_2 = \lg^* n < \lg f_{13}$$

So,

$$f_9 > f_{17} > f_{24} > f_{13} > f_2 > f_{14} > f_{25} > f_1 > f_{12} = f_{18}$$