

# Group centrality or network centralization index \*

Bang Ye Wu  
CSIE, Chung Cheng University, Taiwan

October 3, 2008

\*a supplement to my SNA course

We talk the centrality both for actors and a group of actors. The latter can also be regarded as the “network centrality” or “network centrality index”. There are several definitions. Here the author adopt Freeman’s definition but he didn’t describe it clearly in the book. The formal definition can be found in another text-book

S. Wasserman, K. Faust, *Social Network Analysis*, Cambridge University Press, Cambridge, 1994.

## **Group Degree Centralization**

By Freeman’s definition

$$C_D(G) = \frac{\sum_v(\Delta_G - C_D(v))}{\max_H \sum_{v \in H}(\Delta_H - C_D(v))}$$

where  $\Delta_G$  is the maximum degree of any node in  $G$ ,  $C_D(v)$  is the degree of node  $v$  in graph

$G$  and the maximum is taken over all possible graph of the same order (the same number of nodes), i.e., a star. Let  $n$  and  $m$  denote the numbers of nodes and edges, respectively. We can also have

$$C_D(G) = \frac{n\Delta_G - \sum_v(C_D(v))}{(n-1)(n-2)}$$

For an undirected graph,

$$C_D(G) = \frac{n\Delta_G - 2m}{(n-1)(n-2)}$$

For a directed graph,

$$C_D^{in}(G) = \frac{n\Delta_G^{in} - m}{(n-1)(n-2)}$$

and

$$C_D^{out}(G) = \frac{n\Delta_G^{out} - m}{(n-1)(n-2)}$$

For the example in our text book,  $n = 10$ ,  $m = 49$ ,  $\Delta_G^{out} = 8$ ,  $\Delta_G^{in} = 9$ . Therefore

$$C_D^{in}(G) = \frac{10 \times 9 - 49}{9 \times 8} = 41/72 = 56.94\%$$

and

$$C_D^{out}(G) = \frac{10 \times 8 - 49}{9 \times 8} = 31/72 = 43.06\%$$

It is only by accident the two values summed to 100%.

Freeman's definitions for other group centralization indices are similar.

### **Closeness centrality:**

Let  $d(u, v)$  denote the distance from  $u$  to  $v$  and  $D(v) = \sum_u d(v, u)$  be the total distance from  $V$  to all other nodes. The closeness of  $v$  is measured by  $1/D(v)$  and normalized to  $C_C(v) = (n - 1)/D(v)$  since the minimum  $D(v)$  is  $n - 1$ , which happens at the center of a star graph. Using Freeman's definition of group centrality, we have

$$C_C(G) = \frac{\sum_v (C_C(v^*) - C_C(v))}{\max_H \sum_{v \in H} (C_C(v^*) - C_C(v))},$$

in which  $v^*$  is the node of maximum closeness, i.e, the median of  $G$ .

The graph of maximum closeness is also a star. For a star, the closeness of the center is 1, and that of any other node is  $(n - 1)/(2n - 3)$ . Then, we have the maximum closeness of any graph of  $n$  nodes is

$$\left(1 - \frac{(n - 1)}{(2n - 3)}\right) (n - 1) = \frac{(n - 1)(n - 2)}{(2n - 3)}$$

Then,

$$C_C(G) = \frac{\sum_v ((n - 1)/D(v^*) - (n - 1)/D(v))}{\frac{(n-1)(n-2)}{(2n-3)}},$$

For the example in our textbook,

$$C_C(G) = \frac{2.09}{\frac{9 \times 8}{17}} = 49.34\%$$

### **Betweenness centrality:**

Let  $C_B(v)$  denote the betweenness of node  $v$ .

The normalized betweenness is given by

$$C'_B(v) = \frac{C_B(v)}{(n-1)(n-2)/2}$$

To compute the group betweenness centrality, we again compute the one of a star at first. For a star, the center has betweenness  $(n-1)(n-2)/2$  and it is zero for all the others. The group centrality of a star is then  $(n-1)^2(n-2)/2$ . Then, we have

$$\begin{aligned} C_B(G) &= \frac{\sum_v (C_B(v^*) - C_B(v))}{(n-1)^2(n-2)/2} \\ &= \frac{\sum_v (C'_B(v^*) - C'_B(v))}{n-1} \end{aligned}$$

For the example network in our textbook,

$$C_B(G) = 181.03/(9) \times 100\% = 20.11\%$$