Chapter 13

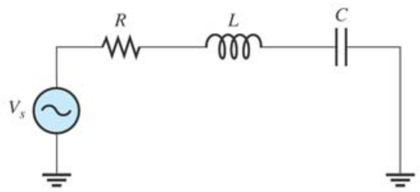
RLC Circuits and Resonance

Objectives

- Determine the impedance of a series RLC circuit
- Analyze series RLC circuits
- Analyze a circuit for series resonance
- Analyze series resonant filters
- Analyze parallel RLC circuits
- Analyze a circuit for parallel resonance
- Analyze the operation of parallel resonant filters

Impedance of Series RLC Circuits

- A series RLC circuit contains both inductance and capacitance
- Since X_L and X_C have opposite effects on the circuit phase angle, the total reactance (X_{tot})is less than either individual reactance



Impedance of Series RLC Circuits

- When $X_L>X_C$, the circuit is predominantly inductive
- When $X_C> X_L$, the circuit is predominantly capacitive
- Total impedance for a series RLC circuit is:

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2}$$

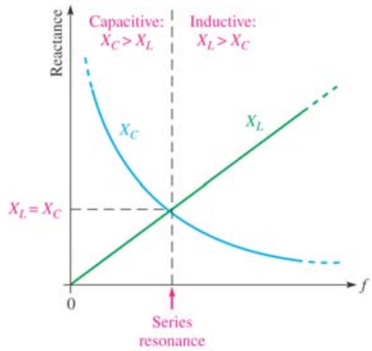
$$\theta = \tan^{-1}(X_{tot}/R)$$

Analysis of Series RLC Circuits

- A series RLC circuit is:
- Capacitive for $X_C > X_L$
- Inductive for $X_L > X_C$
- Resonant for $X_C = X_L$
- At resonance $Z_r = R$
- X_L is a straight line

  $y = mx + b$
- X_C is a hyperbola

  $xy = k$



Voltage Across the Series Combination of L and C

- In a series RLC circuit, the capacitor voltage and the inductor voltage are always 180° out of phase with each other
- Because they are 180° out of phase, V_C and V_L subtract from each other
- The voltage across L and C combined is always less that the larger individual voltage across either element

Series Resonance

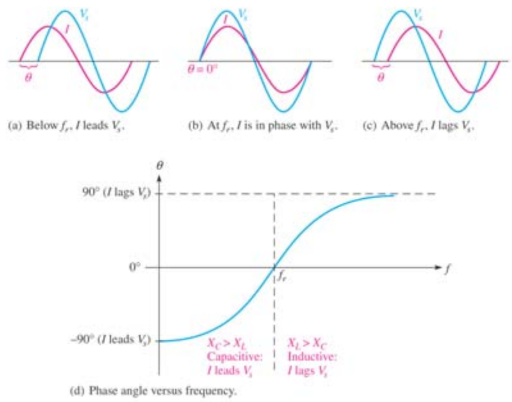
- Resonance is a condition in a series RLC circuit in which the capacitive and inductive reactances are equal in magnitude
- The result is a purely resistive impedance
- The formula for series resonance is:

$$f_r = 1/(2\pi\sqrt{LC})$$

Current and Voltage in a Series RLC Circuit

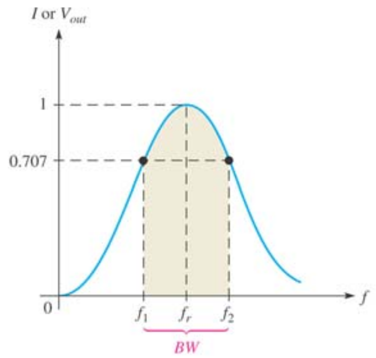
- At the series resonant frequency, the current is maximum ($I_{max} = V_s/R$)
- Above and below resonance, the current decreases because the impedance increases
- At resonance, impedance is equal to R
- The voltages across L and C are maximum at resonance, but they are also equal in magnitude and 180° out of phase, so they cancel (the total voltage across L and C is zero)

Phase Angle of a Series RLC Circuit



Bandwidth of Series Resonant Circuits

- Current is maximum at resonant frequency
- Bandwidth (BW) is the range (f_1 to f_2) of frequencies for which the current is greater than 70.7% of the resonant value



Formula for Bandwidth

- Bandwidth for either series or parallel resonant circuits is the range of frequencies between the upper and lower cutoff frequencies for which the response curve (I or Z) is 0.707 of the maximum value

$$BW = f_2 - f_1$$

- Ideally the center frequency is:

$$f_r = (f_1 + f_2)/2$$

Half-Power Frequencies

- The upper and lower critical frequencies are also called the **half-power frequencies**
  - also called the -3 dB frequencies

$$\text{voltage ratio: } dB = 20 \log (V_{out}/V_{in})$$

$$\text{power ratio: } dB = 10 \log (P_{out}/P_{in})$$

- The true power delivered from the source at these frequencies is one-half the power delivered at the resonant frequency

Selectivity

- Selectivity defines how well a resonant circuit responds to a certain frequency and discriminates against all other frequencies
- The narrower the bandwidth, the greater the selectivity
- The steeper the slope of the response curve, the greater the selectivity

Q Affects Bandwidth

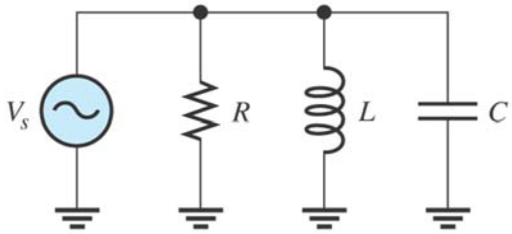
- A higher value of circuit Q (quality) results in a narrower bandwidth
- A lower value of Q causes a wider bandwidth
- The formula for the bandwidth of a resonant circuit in terms of Q is:

$$BW = f_r/Q$$

Impedance of Parallel RLC Circuits

The total impedance of a parallel RLC circuit is:

$$1/\mathbf{Z} = Y = \sqrt{G^2 + B_{tot}^2}$$



Conductance, Susceptance, and Admittance

- Conductance: $\mathbf{G} = 1/R$
- Capacitive Susceptance: $\mathbf{B}_C = 1/X_C$
- Inductive Susceptance: $\mathbf{B}_L = 1/X_L$
- Total Susceptance: $B_{tot} = |B_L - B_C|$
- Admittance: $\mathbf{Y} = \sqrt{G^2 + B_{tot}^2}$
- Total Impedance: $\mathbf{Z}_{tot} = 1/\mathbf{Y}$
- Phase angle: $\theta = \tan^{-1} (\mathbf{B}_{tot}/\mathbf{G})$

Analysis of Parallel RLC Circuits

- The smaller reactance in a parallel circuit dominates because it results in the larger branch current
- At low frequencies $X_L<X_C$; therefore the circuit is inductive
- **Parallel resonance** is the point where $X_L=X_C$
- At high frequencies $X_C<X_L$; therefore the circuit is capacitive

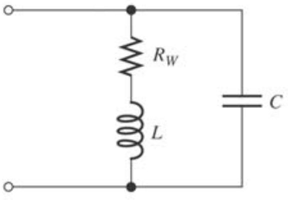
Conversion of Series-Parallel to Parallel

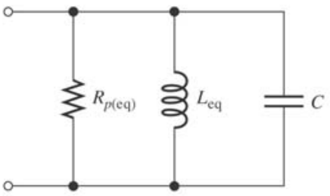
- Equivalent Inductance:

  $L_{eq} = L((Q^2+1)/Q^2)$
- Equivalent Parallel Resistance:

  $R_{p(eq)} = R_W(Q^2+1)$
  - where Q is the quality factor of the coil:

    $Q = X_L/R_W$





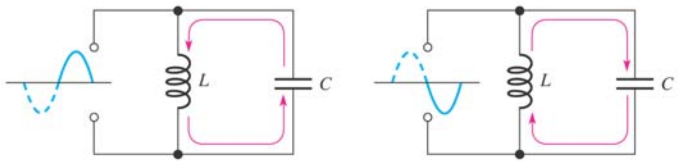
Ideal Parallel Resonance

- Parallel resonance occurs when $X_C = X_L$
- At resonance, the two branch currents are equal in magnitude, and 180° out-of-phase with each other
- I_C and I_L cancel
- Since the total current is zero, the impedance of the ideal parallel LC circuit is infinitely large (∞)
- Ideal (no resistance) parallel resonant frequency:

$$f_r = 1/(2\pi\sqrt{LC})$$

Tank Circuit

- A parallel resonant circuit stores energy in the magnetic field of the coil and the electric field of the capacitor. The energy is transferred back and forth between the coil and capacitor



(a) The coil deenergizes as the capacitor charges.

(b) The capacitor discharges as the coil energizes.

Parallel Resonant Conditions in a Non-ideal Circuit

- A practical treatment of parallel resonant circuits must include the coil resistance
- At parallel resonance:

$$X_{L(eq)} = X_C$$

- The total impedance of the non-ideal tank circuit at resonance can be expressed as the equivalent parallel resistance:

$$Z_r = R_W(Q^2 + 1)$$

Current and Phase Angle at Resonance

- Ideally the total current from the source at resonance is zero because the impedance is infinite
- In the non-ideal case when the coil resistance is considered, there is some total current at the resonant frequency:

$$I_{tot} = V_s/Z_r$$

- Since the impedance is purely resistive at resonance, the phase angle is 0°

Effect of Coil Resistance on the Parallel Resonant Frequency

- Q is the quality factor of the coil, X_L/R_W
- For values of $Q \geq 10$, the parallel resonant frequency is:

$$f_r \cong 1/(2\pi\sqrt{LC})$$

External Parallel Load Resistance

- Often an external load resistance appears in parallel with a tank circuit
- The external resistance effectively appears in parallel with the equivalent parallel resistance ($R_{p(eq)}$) of the coil:

$$R_{p(tot)} = R_L || R_{p(eq)}$$

- The external resistor (R_L) will lower the overall Q, designated Q_O of the circuit:

$$Q_O = R_{p(tot)}/X_{L(eq)}$$

Parallel Resonant Circuits

- For parallel resonant circuits, the impedance is maximum at the resonant frequency
- Total current is minimum at the resonant frequency
- Bandwidth is the same as for the series resonant circuit; the critical frequency impedances are at $0.707Z_{max}$

Summary

- X_L and X_C have opposing effects in a RLC circuit
- In a series RLC circuit, the larger reactance determines the net reactance of the circuit
- At series resonance, the inductive and capacitive reactances are equal
- The impedance of a series RLC circuit is purely resistive at resonance
- In a series RLC circuit, the current is maximum at resonance

Summary

- The reactive voltages V_L and V_C cancel at resonance in a series RLC circuit because they are equal in magnitude and 180° out of phase
- In a parallel RLC circuit, the smaller reactance determines the net reactance of the circuit
- In a parallel resonant circuit, the impedance is maximum at the resonant frequency
- A parallel resonant circuit is commonly called a **tank circuit**

Summary

- The impedance of a parallel RLC circuit is maximum at resonance
- Current is minimum, and ideally equal to zero at resonance
- The phase angle is zero at resonance
- The bandwidth of a parallel resonant circuit is the range of frequencies for which the impedance is $0.707Z_{max}$ or greater

Summary

- The currents in parallel L and C branches are equal in magnitude and 180° out-of-phase with each other and thus they cancel at resonance
- The critical frequencies are the frequencies above and below resonance where the circuit response is 70.7% of the maximum response
- Cutoff frequencies are also called - 3 dB frequencies or critical frequencies
- A higher Q produces a narrower bandwidth