# 11. Other Compression Techniques

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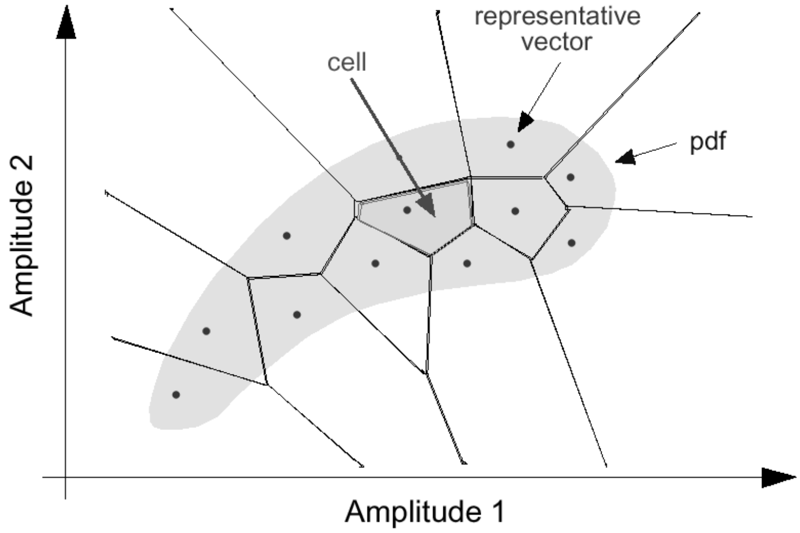
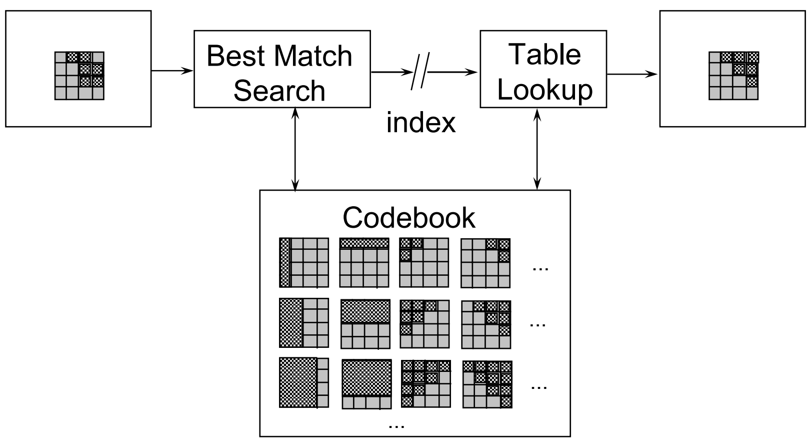
## Non-Standard Coding Techniques

- Vector Quantization (VQ)
- Fractal coding
- Subband and Wavelet coding
- Model-based coding

## Vector Quantization

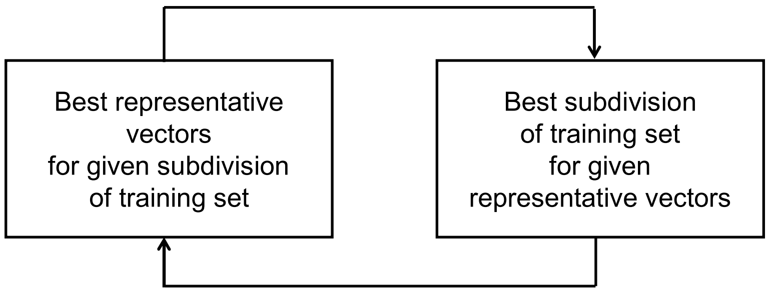



## Vector Quantization (Cont.)



## LBG Algorithm

- Linde, Buzo, Gray, 1980: Lloyd algorithm generalized for VQ




- Assumption: fixed codeword length
- Code book unstructured: full search

## VLC VQ

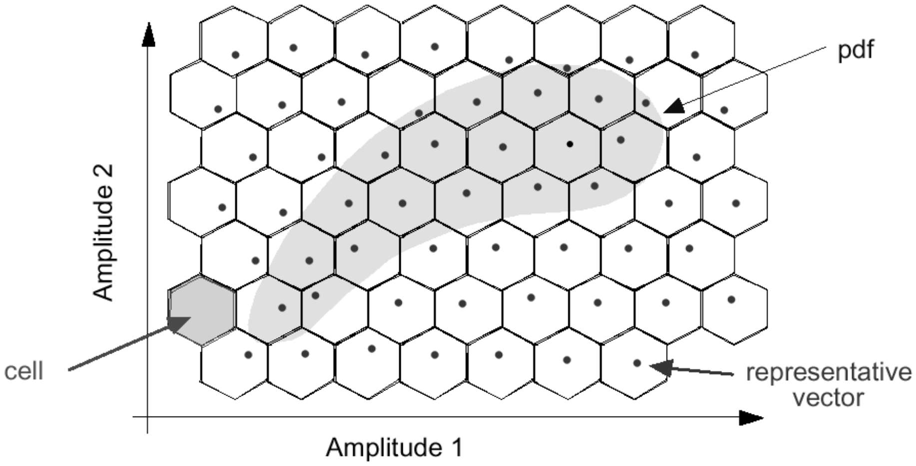
- Chou, Lookabaugh, Gray, 1989: extended LBG algorithm for entropy-coded VQ
- Lagrangian cost function: solve unconstrained problem rather than constrained problem

$$\min\{D+\lambda R\}$$

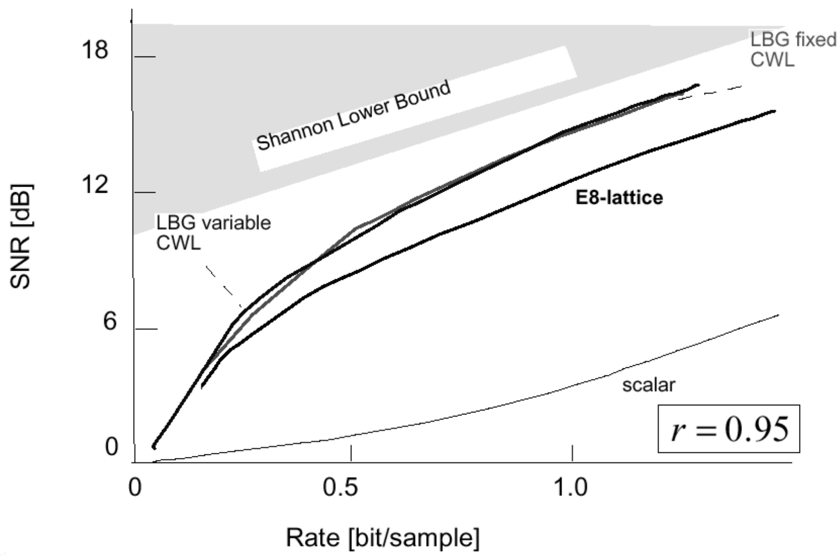
- Unstructured codebook: full search for $\min\{D+\lambda R\}$

| |
|---|
| The most general coder structure: Any source coder can be interpreted as VQ with VLC |

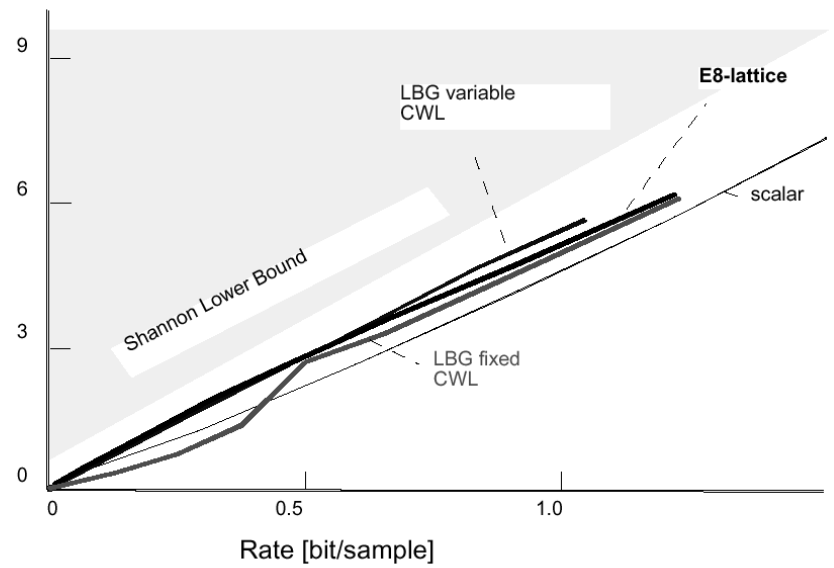
## Lattice Vector Quantization




## 8D VQ of Gauss-Markov Source



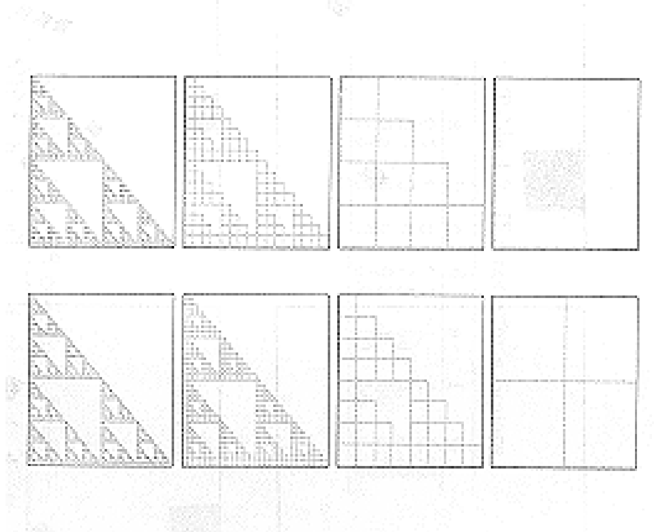
## 8D VQ of Memoryless Laplacian Source




## Vector Quantization

- Highly asymmetrical encoding and decoding complexity
- Encoding complexity depends on the codebook size
- Decoding is done simply by table lookup
- Codebook design usually is an iterative process which involves off-line training
- Different codebook designs result in different quality

## Fractals



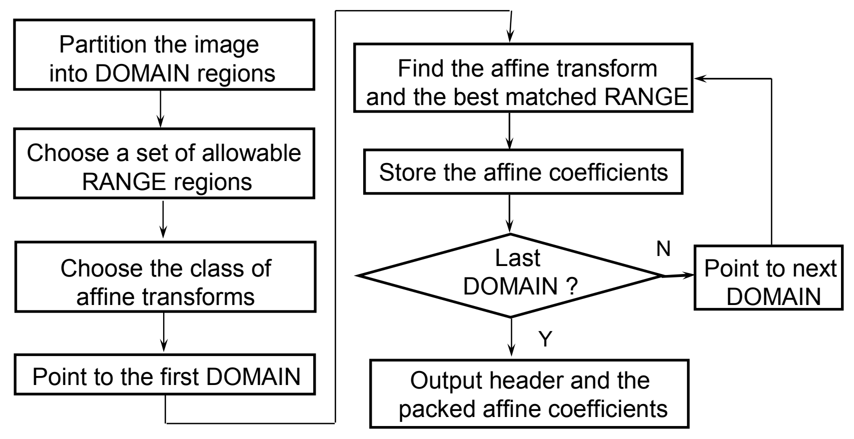
## Affine Transformation

Maps points into new points according to the transform:

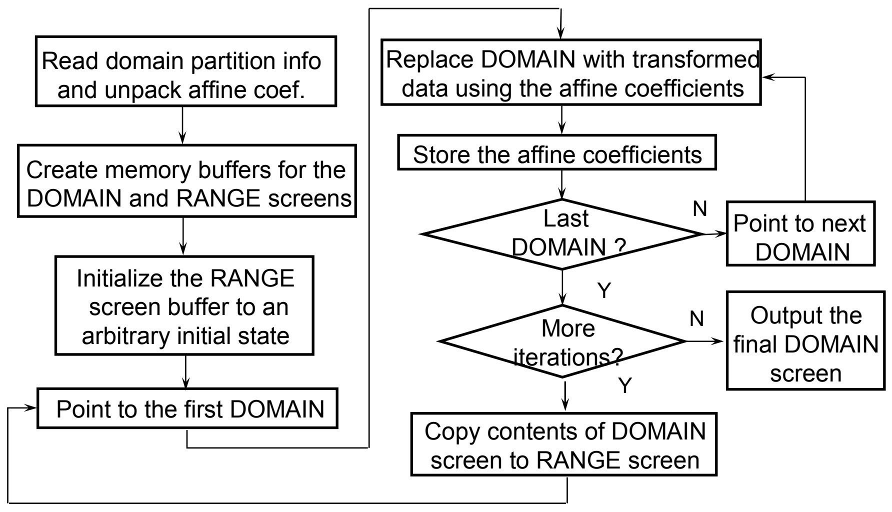
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

- Can model scaling, translation, rotation, etc.

## Fractal Image Coding




## Fractal Image Decoding



## Fractal Coding

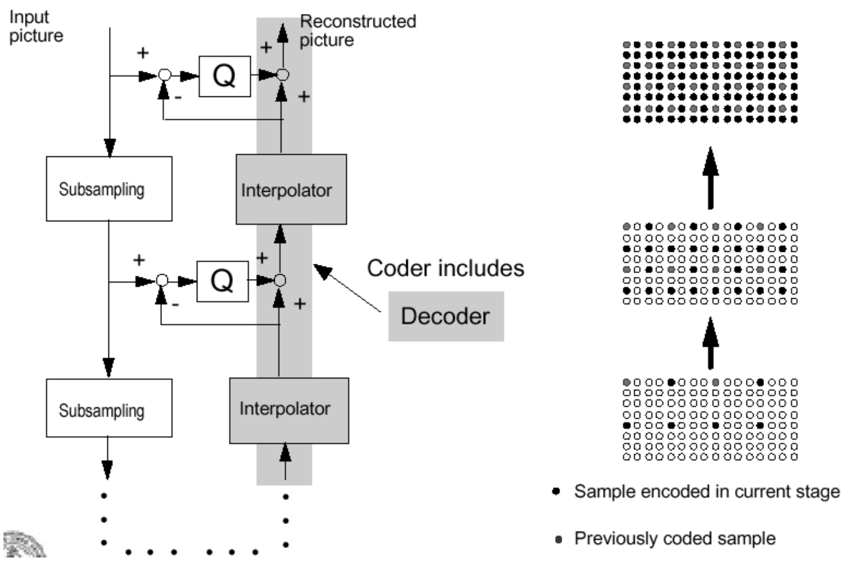
- Real-time encoding is a problem
- Currently quality may not be better than JPEG and MPEG (especially for high quality applications)

\+ Real-time decoding is simple, can be implemented using regular PC

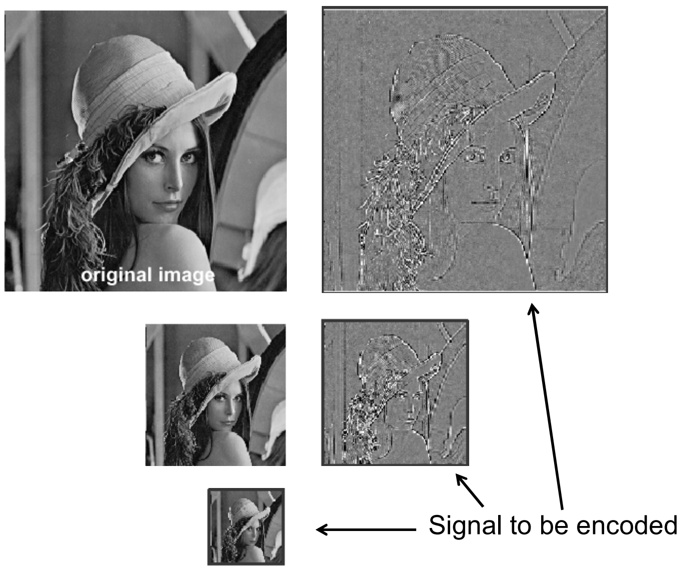
\+ Resolution independent

\+ Relatively new, may have more room for improvement

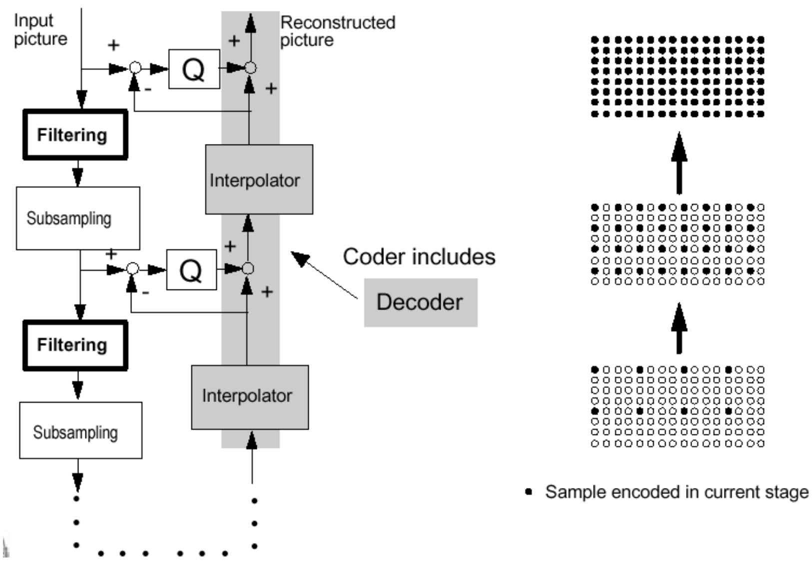
## Interpolation Error Coding



## Interpolation Error Coding (Cont.)



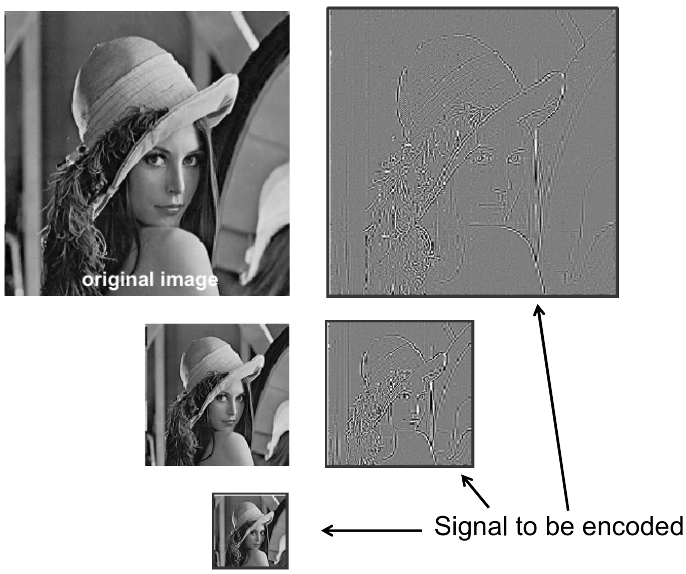

## Predictive Pyramid Coding



## Predictive Pyramid Coding (Cont.)

subsampling factor

## Predictive Pyramid Coding (Cont.)



## Interpolation Error Coding vs. Pyramid

Resolution layer #0, interpolated to original size for display

Interpolation error coding

Pyramid

## Interpolation Error Coding vs. Pyramid (Cont.)

Resolution layer #1, interpolated to original size for display

Interpolation error coding

Pyramid

## Interpolation Error Coding vs. Pyramid (Cont.)

Resolution layer #2, interpolated to original size for display

Interpolation error coding

Pyramid

## Interpolation Error Coding vs. Pyramid (Cont.)

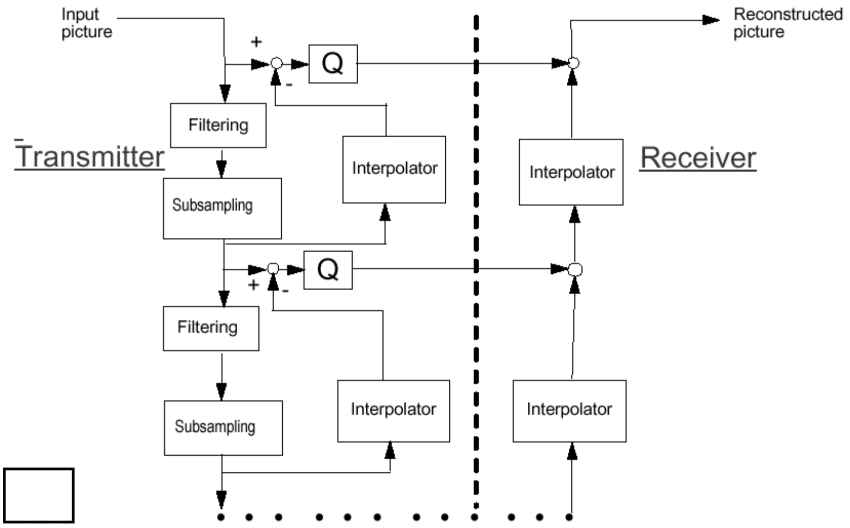
Resolution layer #3

Interpolation error coding

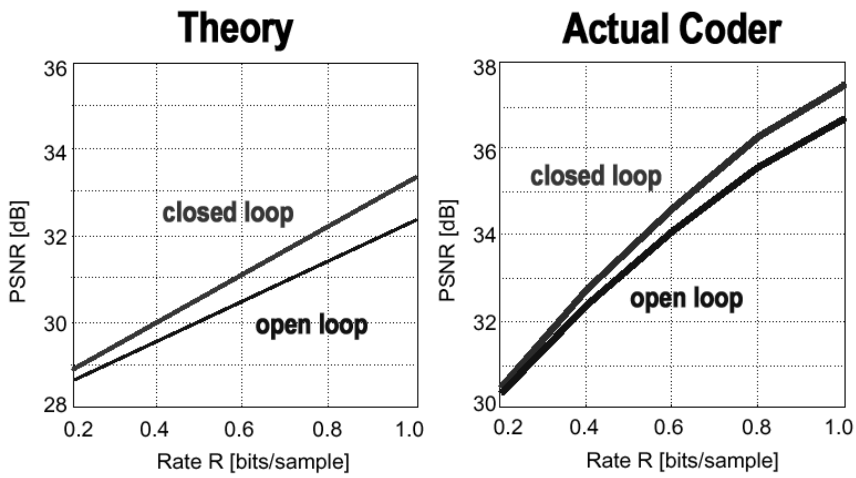
=
(original)

Pyramid

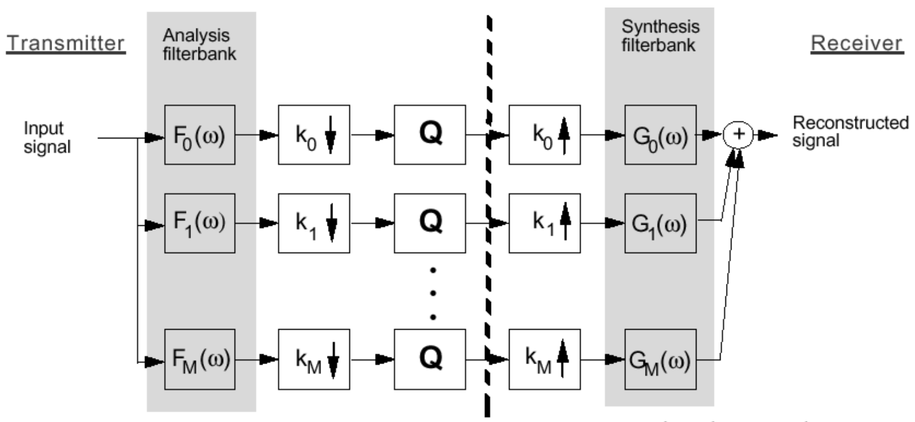
## Open-loop Pyramid (Laplacian Pyramid)




## Two-layer Open- vs. Closed-loop Pyramid

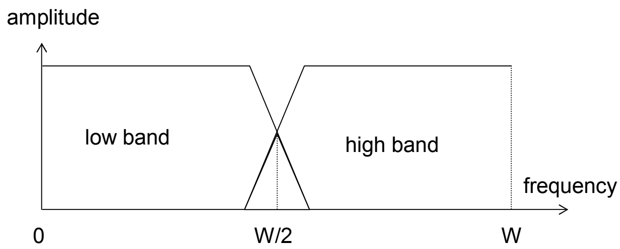


## Subband Coding



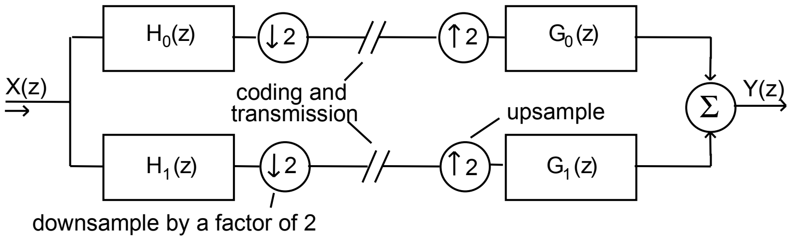

- Number of degrees of freedom is preserved: $\frac{1}{k_0}+\frac{1}{k_1}+...+\frac{1}{k_M}=1$
  *"Critically sampled filter bank"*
- Perfect reconstruction filter bank required

## Two-Band Analysis Filter Band



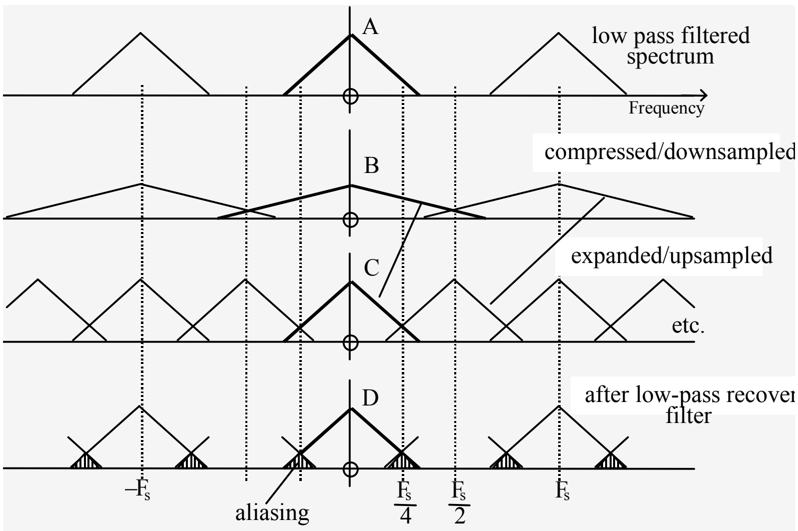

- To eliminate aliasing distortion, the synthesis and analysis filters must have certain relationships

## Two-Band Subband Codec

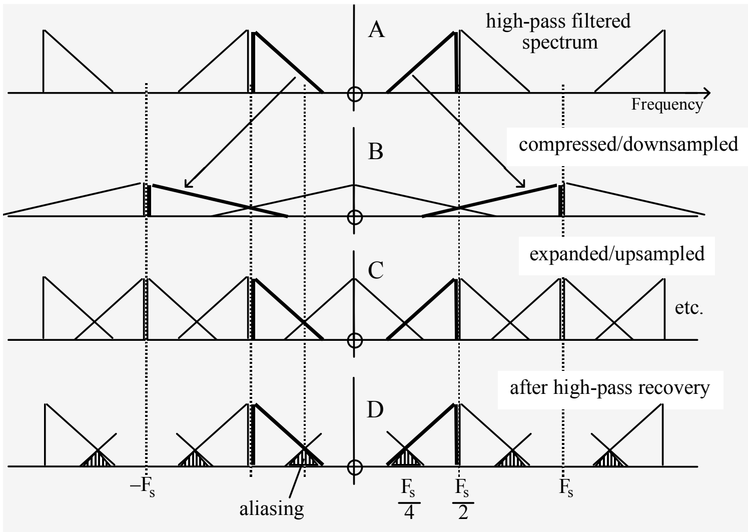



- H0(z) : the z-transform of the low-pass analysis filter
- H1(z) : the z-transform of high-pass analysis filter
- G0(z) and G1(z) are the corresponding synthesis filters
- The downsampling factor is 2, so as the upsampling

## Low-pass Subband Generation and Recovery



## High-pass Subband Generation and Recovery




## Perfect Reconstruction in Subband Coding

The reconstructed output in z-transform notation:

$$Y(z) = G_0(z) \cdot Y_0(z) + G_1(z) \cdot Y_1(z)$$

where $Y_0(z) = \frac{1}{2}[H_0(z) \cdot X(z) + H_0(-z) \cdot X(-z)]$

$$Y_1(z) = \frac{1}{2}[H_1(z) \cdot X(z) + H_1(-z) \cdot X(-z)]$$

where the aliasing components from the downsampling of the lower and higher bands are given by $H_0(-z)X(-z)$ and $H_1(-z)X(-z)$ respectively

Then
$$Y(z) = \frac{1}{2}[H_0(z) \cdot G_0(z) + H_1(z) \cdot G_1(z)]X(z) + \frac{1}{2}[H_0(-z) \cdot G_0(z) + H_1(-z) \cdot G_1(z)]X(-z)$$

## Perfect Reconstruction in Subband Coding

The first term is the desired reconstructed signal, while the second term is aliased components

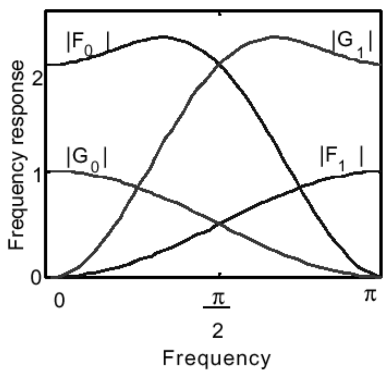
So, sets $G_0(z) = H_1(-z)$ and $G_1(z) = -H_0(-z)$

With such a relation between the synthesis and analysis filters, the reconstructed signal now becomes:

$$Y(z) = \frac{1}{2}[H_0(z) \cdot H_1(-z) - H_0(-z) \cdot H_1(z)]X(z)$$

## Example: Two-channel Filter Bank with Perfect Reconstruction

- Analysis filter impulse responses:
  - Lowpass band:
    $\frac{1}{4}(-1,+2,+6,+2,-1)$
  - Highpass band:
    $\frac{1}{4}(+1,-2,+1)$
- Synthesis filter impulse responses:
  - Lowpass band:
    $\frac{1}{4}(+1,+2,+1)$
  - Highpass band:
    $\frac{1}{4}(+1,+2,-6,+2,+1)$
- Frequency responses:



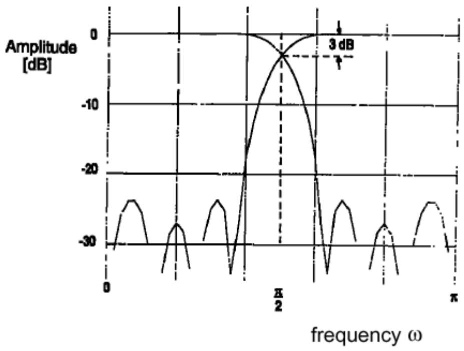
## Quadrature Mirror Filter

- QMFs achieve aliasing cancellation by choosing

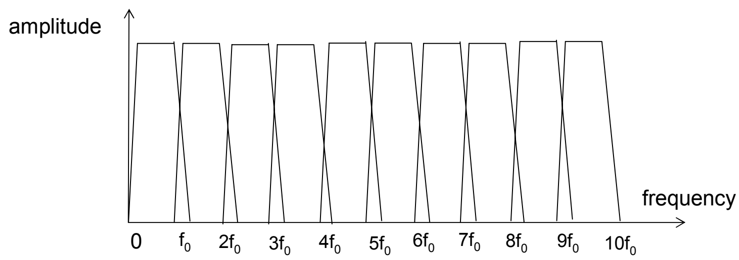
$$F_1(\omega) = F_0(\omega+\pi)$$
$$= -G_1(\omega) = G_0(\omega+\pi)$$

- Highpass band is the mirror image of the lowpass band in the frequency domain

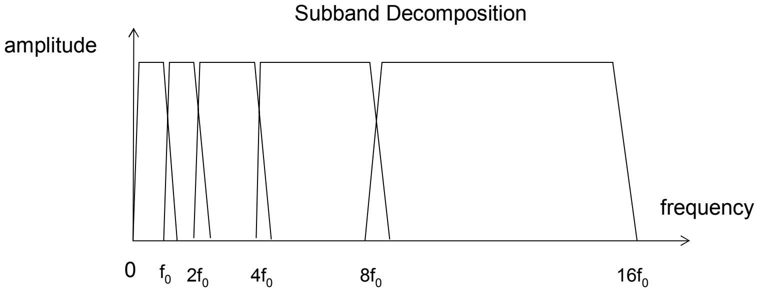
<u>Example:</u>
<u>16-tap QMF filterbank</u>




## Subband and Wavelet Coding

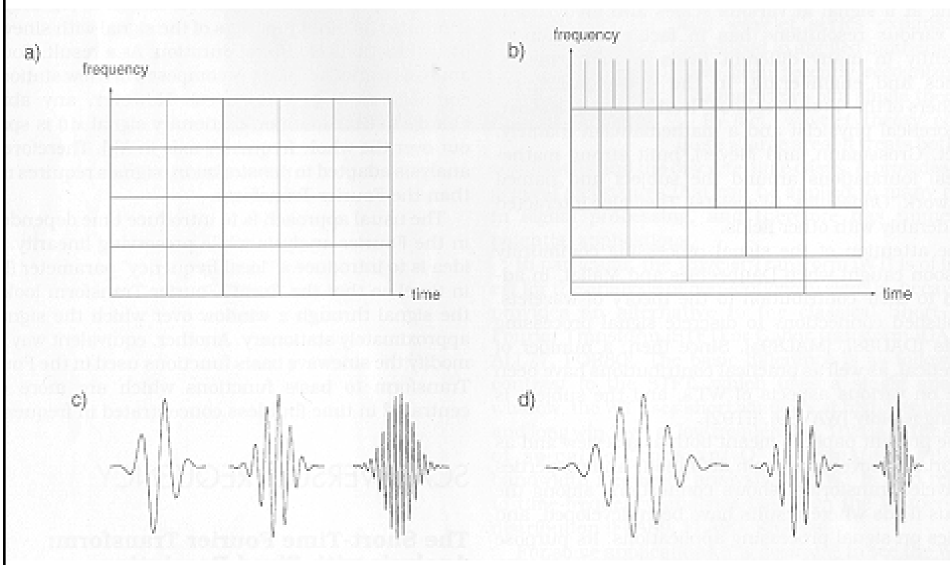



Subband Decomposition

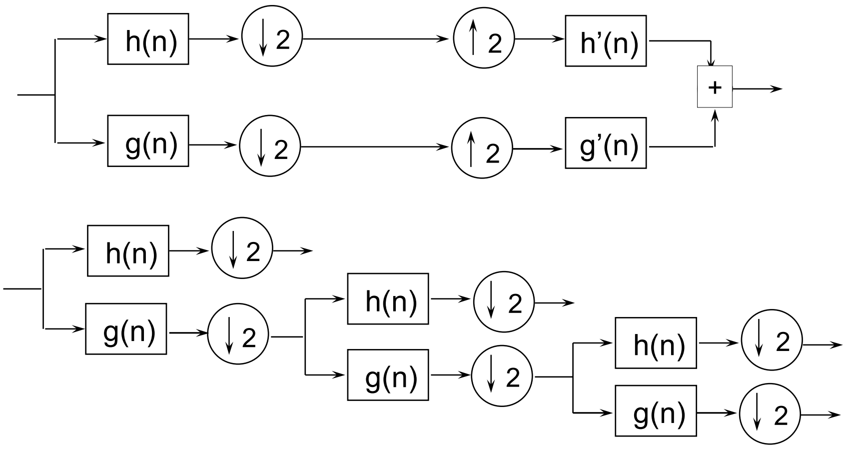



Wavelet Decomposition

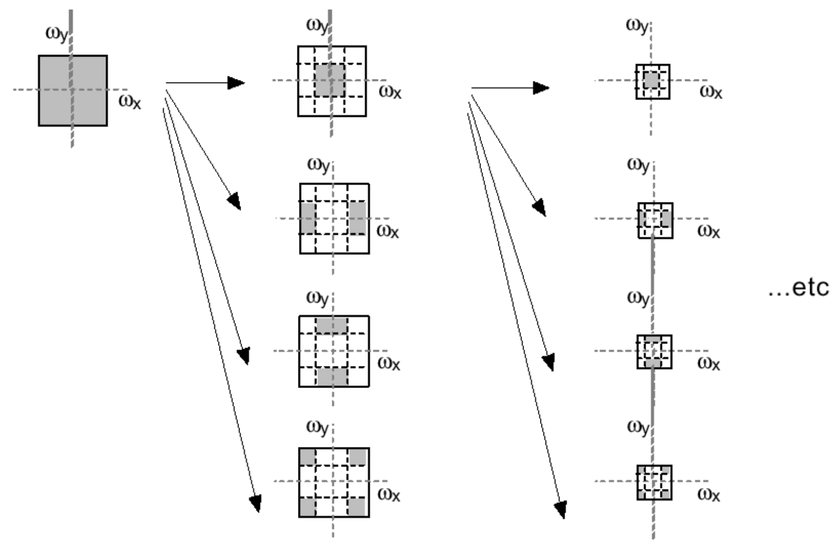
## Subband and Wavelet Coding (Cont.)



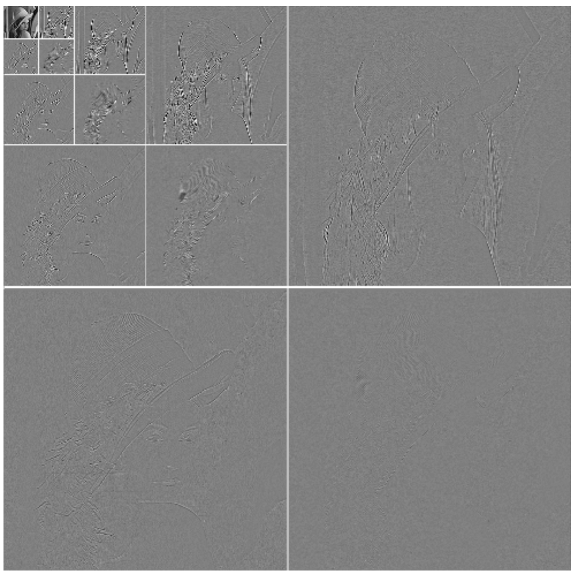

## Subband and Wavelet Coding (Cont.)



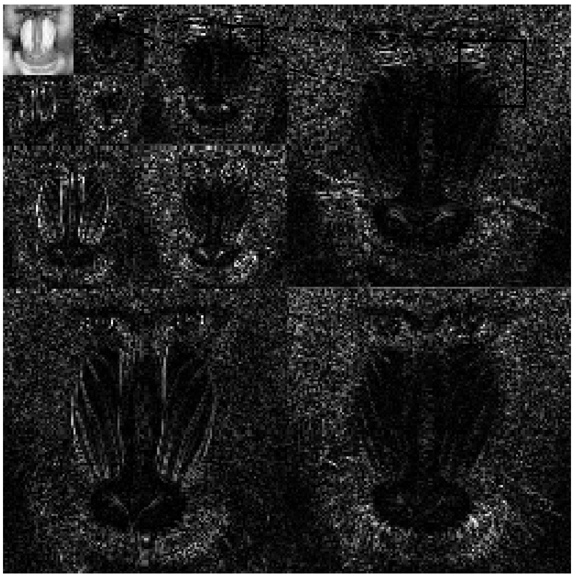
## 2-D Discrete Wavelet Transform




## 2-D Discrete Wavelet Transform (Cont.)



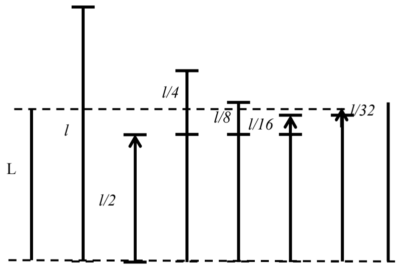
## Similarities among Image Subbands



## Zerotree Coding of Higher Bands

- The higher order wavelet coefficients are coded with the embedded zero-tree wavelet (EZW)
- The method
  - based on the concept of quantization by *successive approximation*,
  - exploits the similarities of the bands of the same orientation.

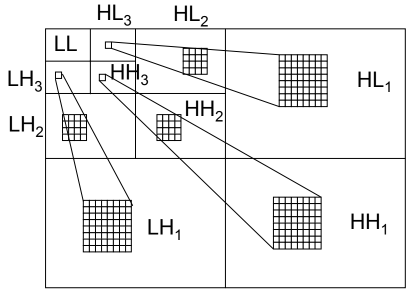
## Principle of Successive Approximation



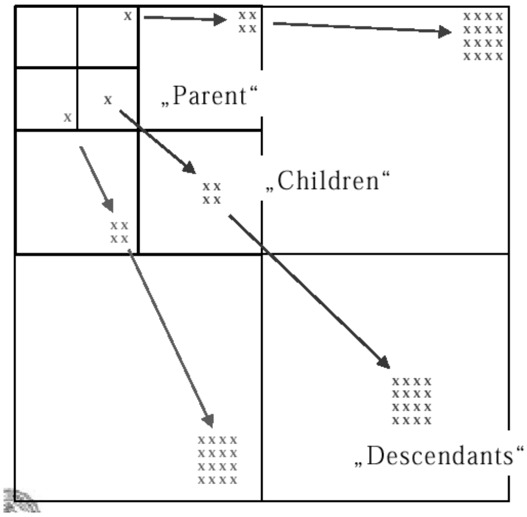

– As shown above, the quantized length, can be expressed as

## Zerotree Coding: Quadtree Representation of Higher Bands

- Subimages of lower bands have quarter dimensions of their higher bands
- A quad-tree representation of the bands of the same orientation for a 10-band splitting is shown below (three-stage wavelet transform)
- If a coefficient in $LH_3$ is zero, it's more likely that its children in higher bands of $LH_2$ and $LH_1$ will also be zero => "*zero tree*"



## Embedded Zerotree Wavelet Algorithm




- Idea: Conditional coding of all descendants (incl. children)
- Coefficient magnitude > threshold: significant coefficients
- Four cases
  - ZTR: zero-tree, coefficient and all descendants are **not** significant
  - IZ: coefficient is not significant, but some descendants are significant
  - POS: positive significant
  - NEG: negative significant

## Embedded Zerotree Wavelet Algorithm (Cont.)

- For the highest bands, ZTR and IZ symbols are merged into one symbol Z
- Successive approximation quantization and encoding
  - Initial "dominant" pass
    - Set initial threshold T, determine significant coefficients
    - Arithmetic coding of symbols ZTR, IZ, POS, NEG
  - Subordinate pass
    - Refine magnitude of coefficients **found significant so far** by one bit (subdivide magnitude bin by two)
    - Arithmetic coding of sequence of zeros and ones.
  - Repeat dominant pass
    - Set previously found significant coefficients to zero
    - Decrease threshold by factor of 2, determine new significant coefficients
    - Arithmetic coding of symbols ZTR, IZ, POS, NEG
  - Repeat subordinate and dominate passes, until bit budget is exhausted.

## A Coding Example of EZW (Shapiro 93)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 63 | -34 | 49 | 10 | 7 | 13 | -12 | 7 |
| -31 | 23 | 14 | -13 | 3 | 4 | 6 | -1 |
| 15 | 14 | 3 | -12 | 5 | -7 | 3 | 9 |
| -9 | -7 | -14 | 8 | 4 | -2 | 3 | 2 |
| -5 | 9 | -1 | 47 | 4 | 6 | -2 | 2 |
| 3 | 0 | -3 | 2 | 3 | -2 | 0 | 4 |
| 2 | -3 | 6 | -4 | 3 | 6 | 3 | 6 |
| 5 | 11 | 5 | -6 | 0 | 3 | -4 | 4 |

## A Coding Example of EZW (Cont.)

| Subband | Coef Value | Symbol | Rec Value |
|---|---|---|---|
| LL3 | 63 | POS | 48 |
| HL3 | -34 | NEG | -48 |
| LH3 | -31 | IZ | 0 |
| HH3 | 23 | ZTR | 0 |
| HL2 | 49 | POS | -48 |
| HL2 | 10 | ZTR | 0 |
| HL2 | 14 | ZTR | 0 |
| HL2 | -13 | ZTR | 0 |
| LH2 | 15 | ZTR | 0 |
| LH2 | 14 | IZ | 0 |
| LH2 | -9 | ZTR | 0 |
| LH2 | -7 | ZTR | 0 |
| HL1 | 7 | Z | 0 |
| HL1 | 13 | Z | 0 |
| HL1 | 3 | Z | 0 |
| HL1 | 4 | Z | 0 |
| LH1 | -1 | Z | 0 |
| LH1 | 47 | POS | 48 |
| LH1 | -3 | Z | 0 |
| LH1 | -2 | Z | 0 |

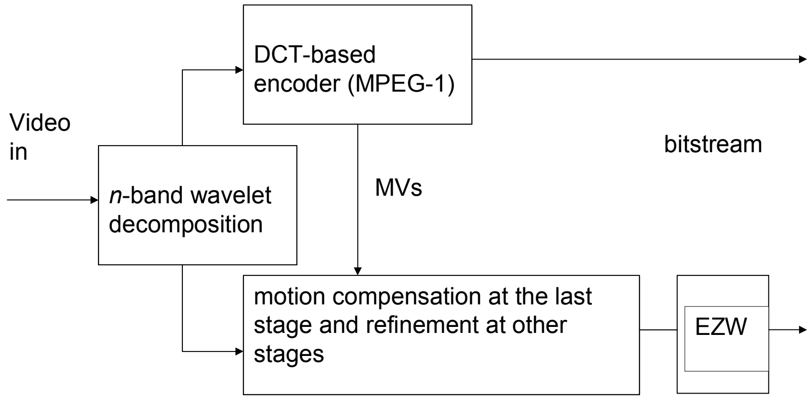
## Features of The EZW Algorithm

- The use of zero trees reduces the number of symbols to be encoded
- The use of a very small alphabet to represent an image makes adaptive arithmetic coding very efficient
- The maximum distortion of each pass is bounded by the threshold used in the pass
- At any given pass, only the coefficients with magnitudes larger than the current threshold are encoded nonzero => prioritize the coefficients of different importance
- With the successive approximation process, the encoding and decoding can stop at any point => not only makes possible an extremely precise bit-rate control, but also achieves the best possible quality at an given bit-budget
- Embedded spatial/SNR scalability

## Embedded Zerotree Wavelet Algorithm (Cont.)

- Decoding: bitstream can be truncated to yield a coarser approximation: “embedded” representation
- Further details: *J. M. Shapiro, “Embedded image coding using zerotrees of wavelet coefficients,” IEEE Transactions on Signal Processing, vol. 41, no. 12, pp. 3445-3462, December 1993.*
- Enhancement SPIHT coder: *A. Said, A., W. A. Pearlman, “A new, fast, and efficient image codec based on set partitioning in hierarchical trees,” IEEE Transactions on Circuits and Systems for Video Technology, vol. 63 , pp. 243-250, June 1996.*
- JPEG-2000 standard similar to SPIHT

## Video Coding with Wavelet Transform




## Set Partitioning in Hierarchical Trees (SPIHT)

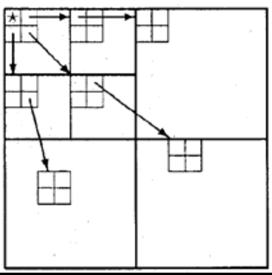
- *A. Said, A., W. A. Pearlman, "A new, fast, and efficient image codec based on set partitioning in hierarchical trees," IEEE T-CSVT, vol. 63 , pp. 243-250, June 1996.*
- JPEG-2000 standard is similar to SPIHT (EBCOT)
- Preliminary Algorithm
  - Output $n$ = floor($\log_2(\max_{(i,j)}\{|C_{i,j}|\})$)
  - Output $\mu_n$, followed by the pixel coordinate $\eta(k)$ and sign of each of the coefficients such that $2^n \le |C_{\eta(k)}| \le 2^{n+1}$ (**sorting pass**)
  - Output the $n$th MSB of all the coefficients with $|C_{i,j}| \ge 2^n$ , in the same order used to send the coordinates (**refinement pass**)
  - Decrease $n$ by one and go to step (2)

## Set Partitioning in Hierarchical Trees (SPIHT)

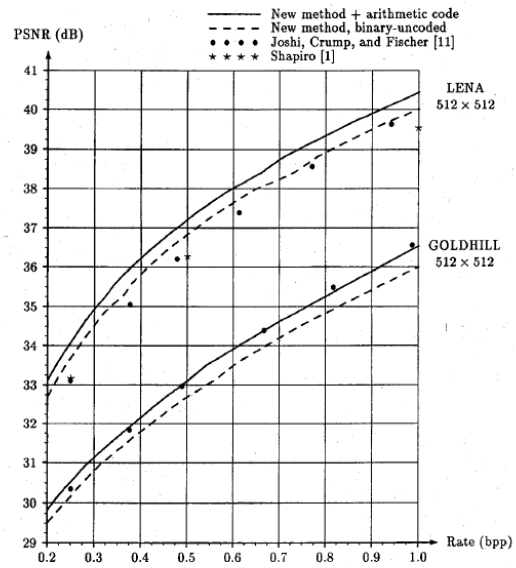
- Function used in the sorting pass
- Assume $O(i,j)$ represents a set of coordinates of all off-spring of node $(i,j)$, that is $O(i,j) = \{(2i,2j). (2i, 2j+1), (2i+1,2j), (2i+1,2j+1)\}$
- Define $D(i,j)$ as a set of coordinates of all descendants of the node $(i,j)$, and $H$, a set of coordinates of all spatial orientation tree
- Finally define $L(i,j) = D(i,j) - O(i,j)$

## Set Partitioning Rules

- the sets $\{(i,j)\}$ and $D(i,j)$, for all $(i,j)$ "member of" $\in H$
- If $D(i,j)$ is significant, it is partitioned into $L(i,j)$ plus the four element sets with $(k,l) \in O(i,j)$
- If $L(i,j)$ is significant, it is partitioned into the four sets $D(k,l)$, with $(k,l) \in O(i,j)$
- Each of the four sets now has the format of the original set, and the sample partition can be used recursively



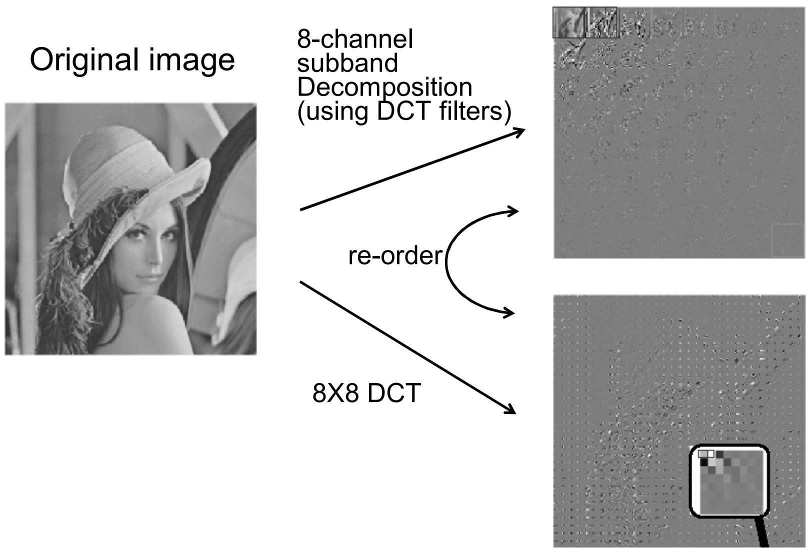
## Performance of SPIHT



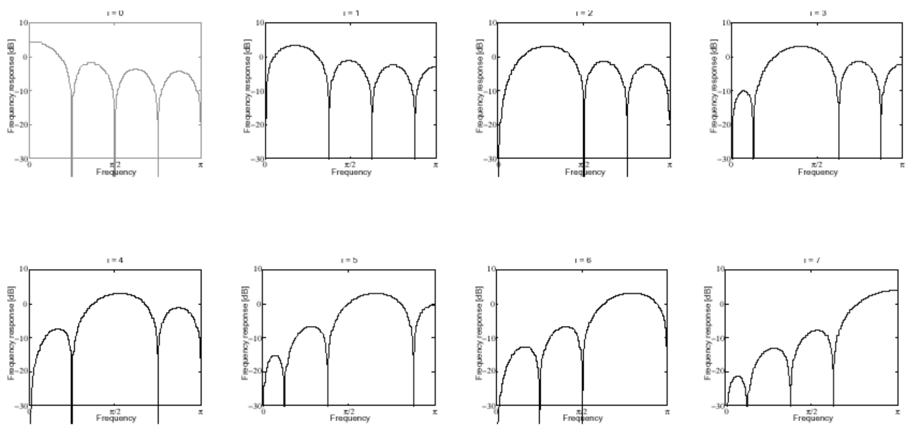

## Subband Coding vs. Transform Coding

- Transform coding is a special case of subband coding with:
  - Number of bands = order of transform *N*
  - Subsampling factor *K* = *N*
  - Length of impulse responses of analysis/synthesis filters ˜ *N*
- Filters used in subband coders are **<u>not</u>** in general orthogonal.

## Subband Coding vs. Transform Coding (Cont.)




## Frequency Response of a DCT of Order N = 8



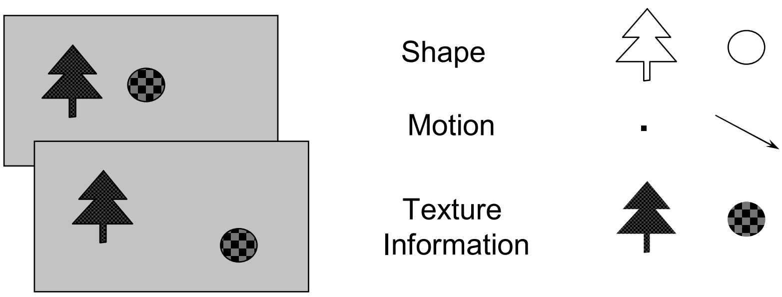
## Model-based Coding

- Has potential to achieve very low bitrates by using source models at both the encoder and decoder

  - Object oriented coding

  - Wired frame model

## Parameters of Object Based Coding



## Wire-Frame Model