Chapter 12

RL Circuits

Objectives

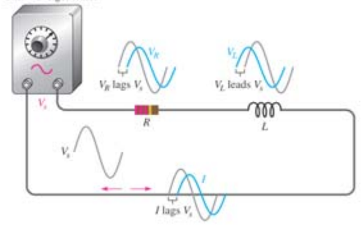
- Describe the relationship between current and voltage in an RL circuit
- Determine impedance and phase angle in a series RL circuit
- Analyze a series RL circuit
- Determine impedance and phase angle in a parallel RL circuit

Objectives

- Analyze a parallel RL circuit
- Analyze series-parallel RL circuits
- Determine power in RL circuits

Sinusoidal Response of RL Circuits

- The inductor voltage leads the source voltage
- Inductance causes a phase shift between voltage and current that depends on the relative values of the resistance and the inductive reactance

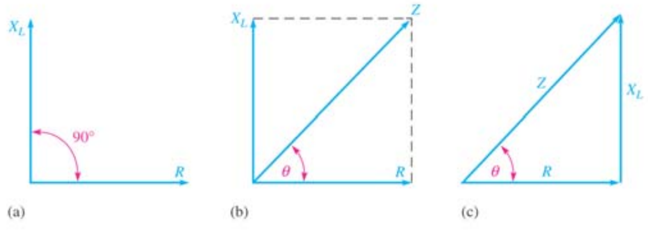


Impedance and Phase Angle of Series RL Circuits

- Impedance of any RL circuit is the total opposition to sinusoidal current and its unit is the ohm
- The phase angle is the phase difference between the total current and the source voltage
- The impedance of a series RL circuit is determined by the resistance (R) and the inductive reactance (X_L)

The Impedance Triangle

- In ac analysis, both R and X_L are treated a phasor quantities, with X_L appearing at a +90° angle with respect to R
- θ is the phase angle between applied voltage and current



The Impedance Triangle

- The impedance magnitude of the series RL circuit in terms of resistance and reactance:

$$Z = \sqrt{R^2 + X^2_L}$$

  - The magnitude of the impedance (Z) is expressed in ohms
- The phase angle is:

$$\theta = \tan^{-1}(X_L/R)$$

Ohm's Law

- Application of Ohm's Law to series RL circuits involves the use of the phasor quantities **Z**, **V**, and **I**

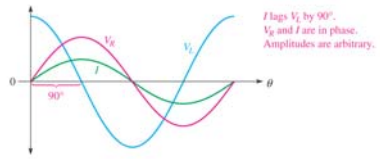
$$\mathbf{V = IZ}$$

$$\mathbf{I = V/Z}$$

$$\mathbf{Z = V/I}$$

Relationships of the Current and Voltages in a Series RL Circuit

- Resistor voltage is in phase with the current
- Inductor voltage leads the current by 90°
- There is a phase difference of 90° between the resistor voltage, V_R, and the inductor voltage, V_L



Kirchhoff's Voltage Law

- From KVL, the sum of the voltage drops must equal the applied voltage
- The magnitude of the source voltage is:

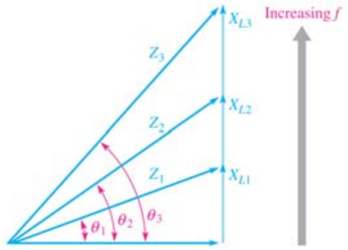
$$V_s = \sqrt{V^2_R + V^2_L}$$

- The phase angle between resistor voltage and source voltage is:

$$\theta = \tan^{-1}(V_L/V_R)$$

Variation of Impedance and Phase Angle with Frequency

- Inductive reactance varies directly with frequency
- Z is directly dependent on frequency
- Phase angle θ also varies directly with frequency



Impedance and Phase Angle of Parallel RL Circuits

- The magnitude of the total impedance of a two-component parallel RL circuit is:

$$Z = RX_L / \sqrt{R^2 + X^2_L}$$

- The phase angle between the applied voltage and the total current is:

$$\theta = \tan^{-1}(R/X_L)$$

Conductance (G), Susceptance (B), and Admittance (Y)

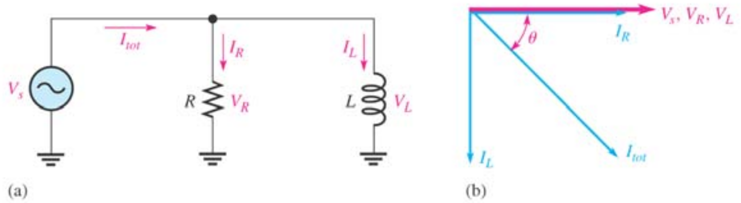
- Conductance is: **G** = 1/R
- Inductive Susceptance is: $\mathbf{B}_L = 1/X_L$
- Admittance is: **Y** = 1/Z
- Total admittance is the phasor sum of conductance and the inductive susceptance:

$$\mathbf{Y} = \sqrt{G^2 + B_L^2}$$

The unit for G, B_L an Y is siemens (S)

Analysis of Parallel RL Circuits

- The total current, I_{tot}, divides at the junction into the two branch currents, I_R and I_L
- V_s, V_R, and V_L are all in phase and of the same magnitude



Kirchhoff's Current Law

- The current through the inductor lags the voltage and the resistor current by 90°
- By Kirchhoff's Current Law, the total current is the phasor sum of the two branch currents:

$$I_{tot} = \sqrt{I^2_R + I^2_L}$$

- Phase angle: $\theta = \tan^{-1}(I_L/I_R)$

Series Parallel RL Circuits

- A first approach to analyzing circuits with combinations of both series and parallel R and L elements is to:
  - Find the series equivalent resistance ($R_{(eq)}$) and inductive reactance ($X_{L(eq)}$) for the parallel portion of the circuit
  - Add the resistances to get the total resistance and add the reactances to get the total reactance
  - Determine the total impedance

Series Parallel RL Circuits

- A second approach to analyzing circuits with combinations of both series and parallel R and L elements is to:
  - Calculate the magnitudes of inductive reactance (X_L)
  - Determine the impedance of each branch
  - Calculate each branch current in polar form
  - Use Ohm's law to get element voltages

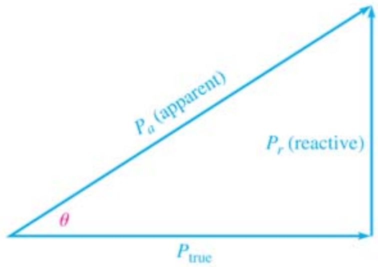
Power in RL Circuits

- When there is both resistance and inductance, some of the energy is alternately stored and returned by the inductance and some is dissipated by the resistance
- The amount of energy converted to heat is determined by the relative values of the resistance and the inductive reactance
- The Power in the inductor is reactive power:

$$P_r = I^2X_L$$

Power Triangle for RL Circuits

- The apparent power (P_a) is the resultant of the average power (P_{true}) and the reactive power (P_R)
- Recall Power Factor: PF = cos θ



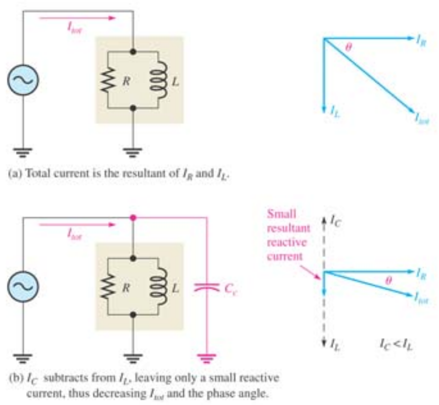
Significance of the Power Factor

- Many practical loads have inductance as a result of their particular function, and it is essential for their proper operation
- Examples are: transformers, electric motors and speakers
- A higher power factor is an advantage in delivering power more efficiently to a load

Power Factor Correction

- Power factor of an inductive load can be increased by the addition of a capacitor in parallel
  - The capacitor compensates for the the phase lag of the total current by creating a capacitive component of current that is 180° out of phase with the inductive component
  - This has a canceling effect and reduces the phase angle (and power factor) as well as the total current, as illustrated on next slide

Power Factor Correction



(a) Total current is the resultant of I_R and I_L.

(b) I_C subtracts from I_L, leaving only a small reactive current, thus decreasing I_{tot} and the phase angle.

RL Circuit as a Low-Pass Filter

- An inductor acts as a short to dc
- As the frequency is increased, so does the inductive reactance
  - As inductive reactance increases, the output voltage across the resistor decreases
  - A series RL circuit, where output is taken across the resistor, finds application as a low-pass filter

RL Circuit as a High-Pass Filter

- For the case when output voltage is measured across the inductor
  - At dc, the inductor acts a short, so the output voltage is zero
  - As frequency increases, so does inductive reactance, resulting in more voltage being dropped across the inductor
  - The result is a high-pass filter

Summary

- When a sinusoidal voltage is applied to an RL circuit, the current and all the voltage drops are also sine waves
- Total current in an RL circuit always lags the source voltage
- The resistor voltage is always in phase with the current
- In an ideal inductor, the voltage always leads the current by 90°

Summary

- In an RL circuit, the impedance is determined by both the resistance and the inductive reactance combined
- Impedance is expressed in units of ohms
- The impedance of an RL circuit varies directly with frequency
- The phase angle (θ) if a series RL circuit varies directly with frequency

Summary

- You can determine the impedance of a circuit by measuring the source voltage and the total current and then applying Ohm's law
- In an RL circuit, part of the power is resistive and part reactive
- The phasor combination of resistive power and reactive power is called *apparent power*
- The power factor indicates how much of the apparent power is true power

Summary

- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit
- In an RL lag network, the output voltage lags the input voltage in phase
- In an RL lead network, the output voltage leads the input voltage in phase
- A filter passes certain frequencies and rejects others