Chapter 10

RC Circuits

Objectives

- Describe the relationship between current and voltage in an RC circuit
- Determine impedance and phase angle in a series RC circuit
- Analyze a series RC circuit
- Determine the impedance and phase angle in a parallel RC circuit

Objectives

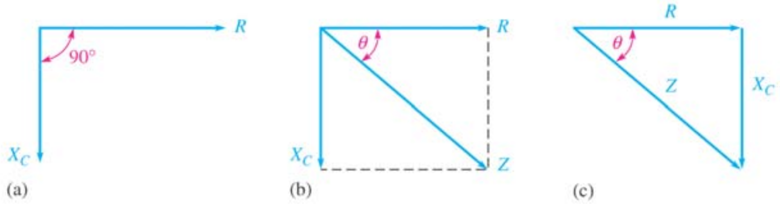
- Analyze a parallel RC circuit
- Analyze series-parallel RC circuits
- Determine power in RC circuits

Sinusoidal Response of RC Circuits

- When a circuit is purely resistive, the phase angle between applied voltage and total current is zero
- When a circuit is purely capacitive, the phase angle between applied voltage and total current is 90°
- When there is a combination of both resistance and capacitance in a circuit, the phase angle between the applied voltage and total current is somewhere between 0° and 90°, depending on relative values of resistance and capacitance

Impedance and Phase Angle of Series RC Circuits

- In the series RC circuit, the total impedance is the phasor sum of R and jX_C
- Impedance magnitude: $Z = \sqrt{R^2 + X^2_C}$
- Phase angle: $\theta = \tan^{-1}(X_C/R)$



Analysis of Series RC Circuits

- The application of Ohm's law to series RC circuits involves the use of the quantities Z, V, and I as:

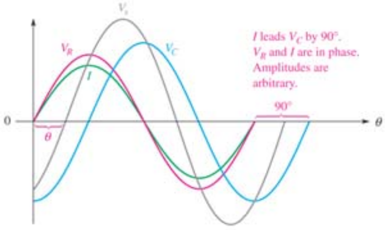
$$\mathbf{V = IZ}$$

$$\mathbf{I = V/Z}$$

$$\mathbf{Z = V/I}$$

Relationships of I and V in a Series RC Circuit

- In a series circuit, the current is the same through both the resistor and the capacitor
- The resistor voltage is in phase with the current, and the capacitor voltage lags the current by 90°



KVL in a Series RC Circuit

- From KVL, the sum of the voltage drops must equal the applied voltage (V_S)
- Since V_R and V_C are 90° out of phase with each other, they must be added as phasor quantities
- Magnitude of source voltage:

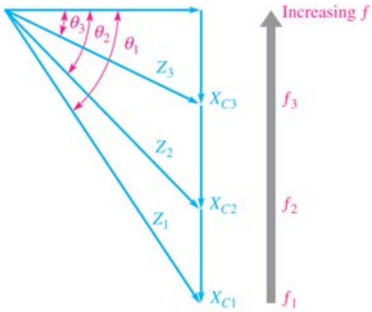
$$V_S = \sqrt{V^2_R + V^2_C}$$

- Phase angle between resistor and source voltages:

$$\theta = \tan^{-1}(V_C/V_R)$$

Variation of Impedance and Phase Angle with Frequency

- For a series RC circuit; as frequency increases:
  - X_C decreases
  - Z decreases
  - θ decreases
  - R remains constant



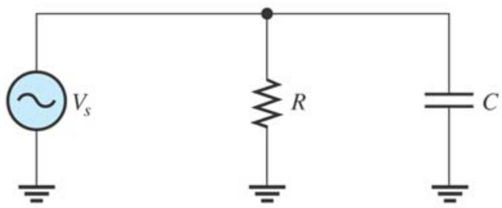
Impedance and Phase Angle of Parallel RC Circuits

- Total impedance :

$$Z = (RX_C) / (\sqrt{R^2 + X^2_C})$$

- Phase angle:

$$\theta = \tan^{-1}(R/X_C)$$



Conductance, Susceptance and Admittance

- Conductance is the reciprocal of resistance:

$$G = 1/R$$

- Capacitive susceptance is the reciprocal of capacitive reactance:

$$B_C = 1/X_C$$

- Admittance is the reciprocal of impedance:

$$Y = 1/Z$$

Ohm's Law

- Application of Ohm's Law to parallel RC circuits using impedance can be rewritten for admittance (Y=1/Z):

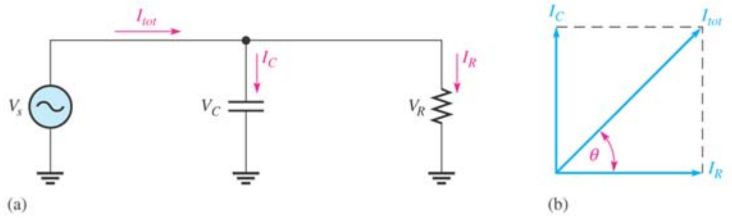
$$\mathbf{V = I/Y}$$

$$\mathbf{I = VY}$$

$$\mathbf{Y = I/V}$$

Relationships of the Currents and Voltages in a Parallel RC Circuit

- The applied voltage, V_S, appears across both the resistive and the capacitive branches
- Total current I_{tot}, divides at the junction into the two branch current, I_R and I_C



Kirchhoff's Current Law

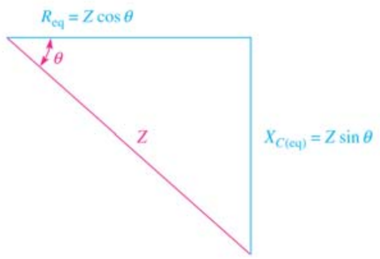
- Current through the resistor is in phase with the voltage
- Current through the capacitor leads the voltage, and thus the resistive current by 90°
- Total current is the phasor sum of the two branch currents
- Magnitude of total current is:

$$I_{tot} = \sqrt{I^2_R + I^2_C}$$

- Phase angle: $\theta = \tan^{-1}(I_C/I_R)$

Conversion from Parallel to Series Form

- For every parallel RC circuit there is an equivalent series RC circuit for any given frequency
- Equivalent resistance and capacitive reactance are indicated on the impedance triangle



Series-Parallel RC Circuits

- An approach to analyzing circuits with combinations of both series and parallel R and C elements is to:
  – Calculate the magnitudes of capacitive reactances (X_C)
  – Find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance

Power in RC Circuits

- When there is both resistance and capacitance, some of the energy is alternately stored and returned by the capacitance and some is dissipated by the resistance
- The amount of energy converted to heat is determined by the relative values of the resistance and the capacitive reactance

Power Triangle for RC Circuits

- The Power can be written as:

$$P_{true} = V_s I_{total} \cos\theta$$

where: $\theta = 0^\circ$ for a purely resistive circuit

since $\cos(0^\circ) = 1$, $P_{true} = V_s I_{total}$

$\theta = 90^\circ$ for a purely capacitive circuit

since $\cos(90^\circ) = 0$, P_{true} = zero

Power Factor

- The term cos θ, in the previous slide, is called the power factor:

$$PF = \cos\theta$$

- The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit
- In an RC circuit, the power factor is referred to as a leading power factor because the current leads the voltage

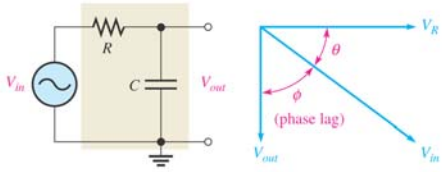
Significance of Apparent Power

- Apparent power is the power that appears to be transferred between the source and the load
- Apparent power consists of two components; a true power component, that does the work, and a reactive power component, that is simply power shuttled back and forth between source and load
- Apparent power is expressed in volt-amperes (VA)

RC Lag Network

- The RC lag network is a phase shift circuit in which the output voltage lags the input voltage

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$



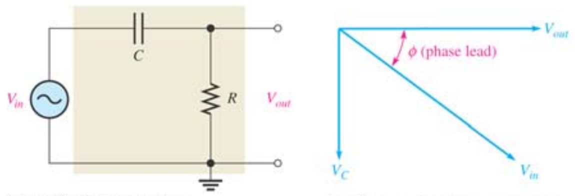
(a) A basic *RC* lag network

(b) Phasor voltage diagram showing the phase lag between V_{in} and V_{out}

RC Lead Network

- The RC lead network is a phase shift circuit in which the output voltage leads the input voltage

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$



(a) A basic *RC* lead network

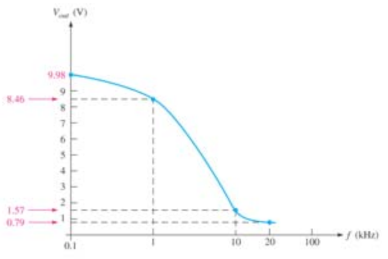
(b) Phasor voltage diagram showing the phase lead between V_{in} and V_{out}

Frequency Selectivity of RC Circuits

- Frequency-selective circuits permit signals of certain frequencies to pass from the input to the output, while blocking all others
- A **low-pass circuit** is realized by taking the output across the capacitor, just as in a lag network
- A **high-pass circuit** is implemented by taking the output across the resistor, as in a lead network

Frequency Selectivity of RC Circuits

- The frequency at which the capacitive reactance equals the resistance in a low-pass or high-pass RC circuit is called the **cutoff frequency:**



$$f_c = 1/(2\pi RC)$$

Summary

- When a sinusoidal voltage is applied to an RC circuit, the current and all the voltage drops are also sine waves
- Total current in an RC circuit always leads the source voltage
- The resistor voltage is always in phase with the current
- The capacitor voltage always lags the current by 90°

Summary

- In an RC circuit, the impedance is determined by both the resistance and the capacitive reactance combined
- Impedance is expressed in units of ohms
- The circuit phase angle is the angle between the total current and the source voltage
- The impedance of a series RC circuit varies inversely with frequency

Summary

- The phase angle (θ) of a series RC circuit varies inversely with frequency
- For each parallel RC circuit, there is an equivalent series circuit for any given frequency
- The impedance of a circuit can be determined by measuring the applied voltage and the total current and then applying Ohm's law

Summary

- In an RC circuit, part of the power is resistive and part is reactive
- The phasor combination of resistive power and reactive power is called apparent power
- Apparent power is expressed in volt-amperes (VA)
- The power factor indicates how much of the apparent power is true power

Summary

- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit
- In a lag network, the output voltage lags the input voltage in phase
- In a lead network, the output voltage leads the input voltage
- A filter passes certain frequencies and rejects others