

Signal Processing First

Lecture 10 FIR Filtering Intro

8/22/2003

© 2003, JH McClellan & RW Schaffer

1

READING ASSIGNMENTS

- This Lecture:
 - Chapter 5, Sects. 5-1, 5-2 and 5-3 (partial)
- Other Reading:
 - Recitation: Ch. 5, Sects 5-4, 5-6, 5-7 and 5-8
 - CONVOLUTION
 - Next Lecture: Ch 5, Sects. 5-3, 5-5 and 5-6

8/22/2003

© 2003, JH McClellan & RW Schaffer

3

LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
 - Weighted Average
 - Running Average
- FINITE IMPULSE RESPONSE FILTERS
 - **FIR** Filters
 - Show how to compute the output $y[n]$ from the input signal, $x[n]$

8/22/2003

© 2003, JH McClellan & RW Schaffer

4

DIGITAL FILTERING



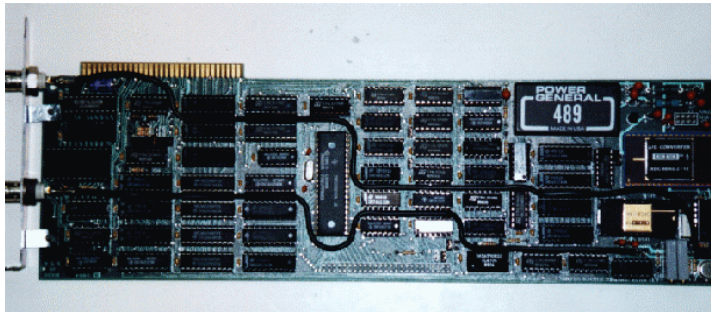
- CONCENTRATE on the COMPUTER
 - PROCESSING ALGORITHMS
 - SOFTWARE (MATLAB)
 - HARDWARE: DSP chips, VLSI
- **DSP**: DIGITAL SIGNAL PROCESSING

8/22/2003

© 2003, JH McClellan & RW Schaffer

5

The TMS32010, 1983



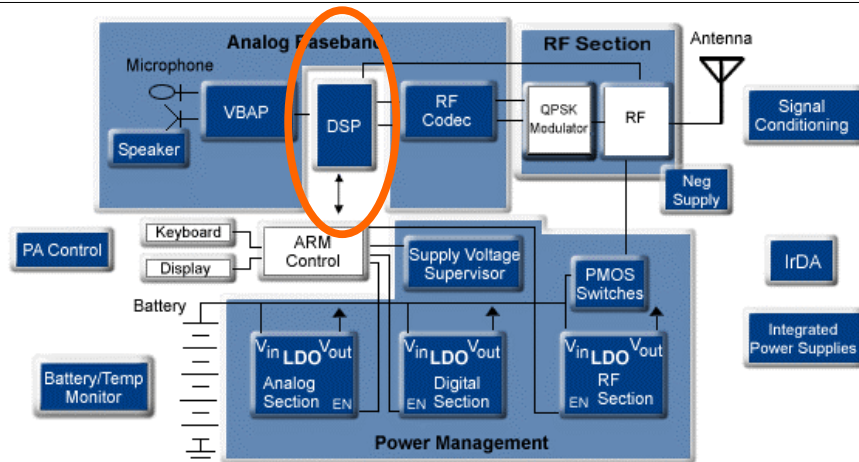
First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971



For the price of a small house, you could have one of these.

Digital Cell Phone (ca. 2000)



Free (?) with 2 year contract

DISCRETE-TIME SYSTEM



- OPERATE on $x[n]$ to get $y[n]$
- WANT a **GENERAL** CLASS of SYSTEMS
 - **ANALYZE** the SYSTEM
 - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
 - **SYNTHESIZE** the SYSTEM

D-T SYSTEM EXAMPLES



- EXAMPLES:
 - POINTWISE OPERATORS
 - SQUARING: $y[n] = (x[n])^2$
 - RUNNING AVERAGE
 - RULE:** “the output at time n is the average of three consecutive input values”

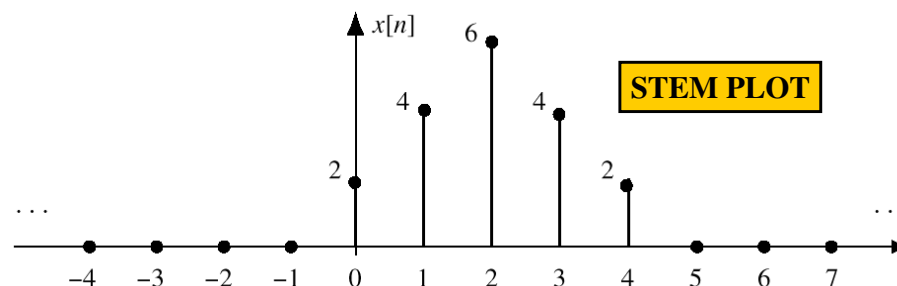
8/22/2003

© 2003, JH McClellan & RW Schafer

10

DISCRETE-TIME SIGNAL

- $x[n]$ is a LIST of NUMBERS
 - INDEXED by “ n ”



8/22/2003

© 2003, JH McClellan & RW Schafer

11

3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
 - Do this for each “ n ”

the following input-output equation

Make a TABLE

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	6	4	2	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

$n=0$ $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

$n=1$ $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

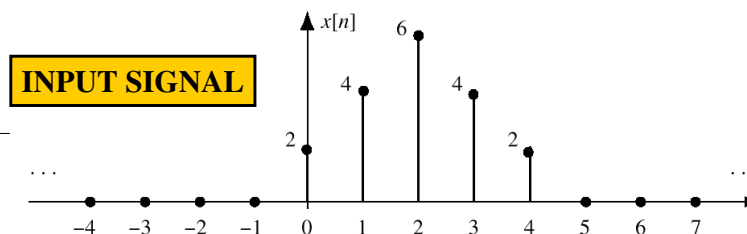
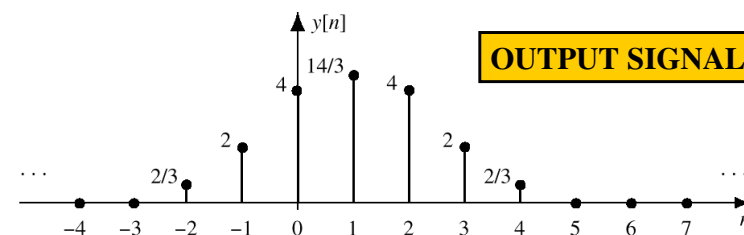


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$



8/22

Figure 5.3 Output of running average, $y[n]$.

13

PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter 123

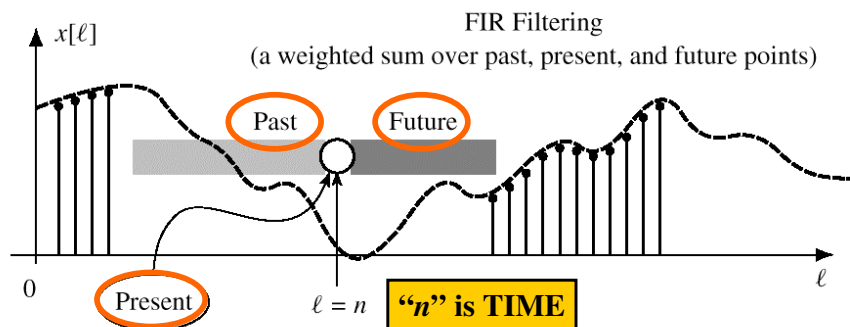


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of $x[n]$
 - IMPORTANT IF "n" represents **REAL TIME**
 - WHEN $x[n]$ & $y[n]$ ARE STREAMS

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

8/22/2003

© 2003, JH McClellan & RW Schafer

15

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

- DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^3 b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

8/22/2003

© 2003, JH McClellan & RW Schafer

16

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **LENGTH** is $L = M+1$
 - NUMBER of FILTER COEFFS is L

8/22/2003

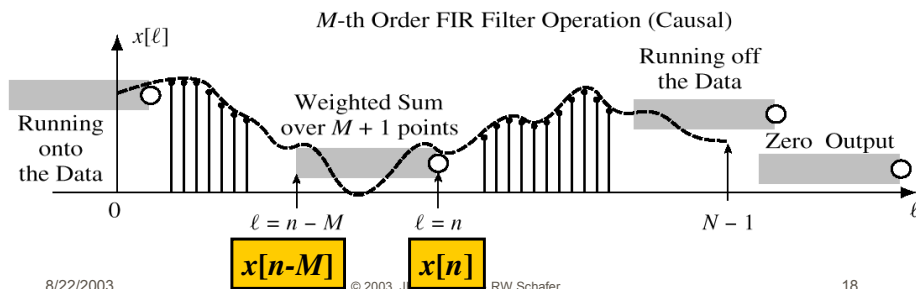
© 2003, JH McClellan & RW Schafer

17

GENERAL FIR FILTER

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

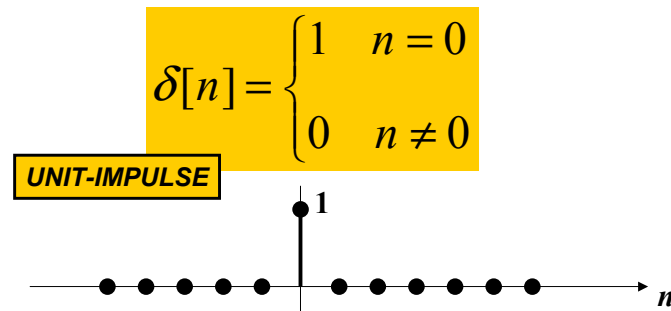


FILTERED STOCK SIGNAL



SPECIAL INPUT SIGNALS

- $x[n] = \text{SINUSOID}$ **FREQUENCY RESPONSE (LATER)**
- $x[n]$ has only one NON-ZERO VALUE



UNIT IMPULSE SIGNAL $\delta[n]$

n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

$\delta[n]$ is NON-ZERO
When its argument
is equal to ZERO

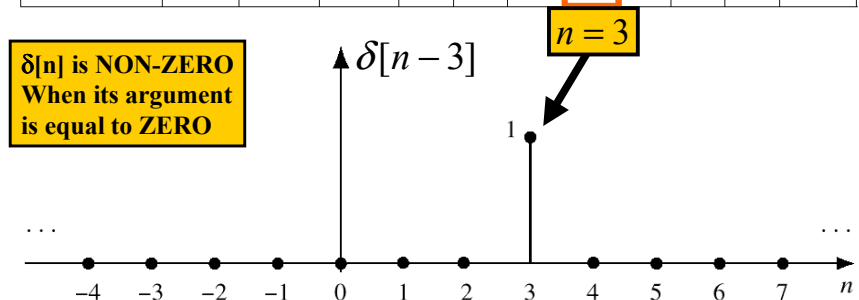
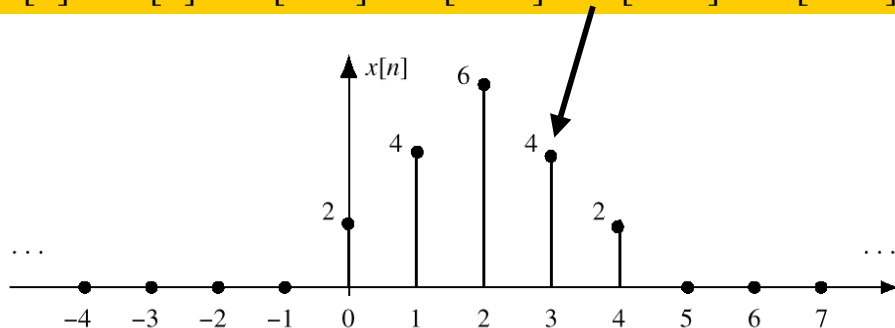


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

MATH FORMULA for $x[n]$

- Use **SHIFTED** IMPULSES to write $x[n]$

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



8/22/2003

© 2003, JH McClellan & RW Schaffer

22

SUM of **SHIFTED** IMPULSES

n	...	-2	-1	0	1	2	3	4	5	6	...
$2\delta[n]$	0	0	0	2	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	4	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	6	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	4	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	2	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_k x[k]\delta[n-k] \quad \leftarrow \text{This formula ALWAYS works}$$

$$= \dots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \dots \quad (5.3.6)$$

4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$

$$x[n] = \delta[n]$$

$$y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$$

- OUTPUT is called **"IMPULSE RESPONSE"**

$$h[n] = \{\dots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \dots\}$$

8/22/2003

© 2003, JH McClellan & RW Schaffer

24

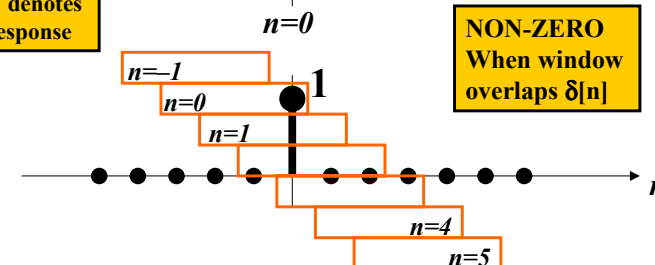
4-pt Avg Impulse Response

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

$\delta[n]$ "READS OUT" the FILTER COEFFICIENTS

$$h[n] = \{\dots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \dots\}$$

"h" in $h[n]$ denotes
Impulse Response



8/22/2003

© 2003, JH McClellan & RW Schaffer

25

FIR IMPULSE RESPONSE

- Convolution = Filter Definition
 - Filter Coeffs = Impulse Response

n	$n < 0$	0	1	2	3	...	M	$M + 1$	$n > M + 1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = \sum_{k=0}^M h[k] x[n-k]$$

CONVOLUTION

8/22/2003

© 2003, JH McClellan & RW Schaefer

26

FILTERING EXAMPLE

- 7-point AVERAGER

- Removes cosine
 - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER

- Changes A slightly

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right) x[n-k]$$

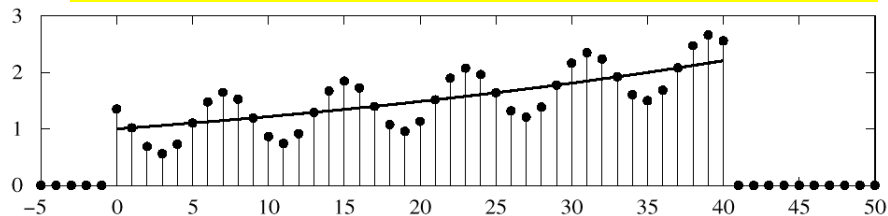
8/22/2003

© 2003, JH McClellan & RW Schaefer

27

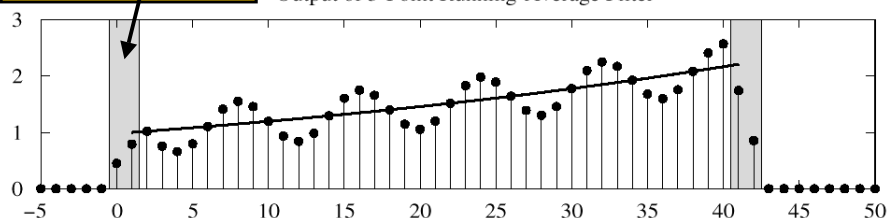
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



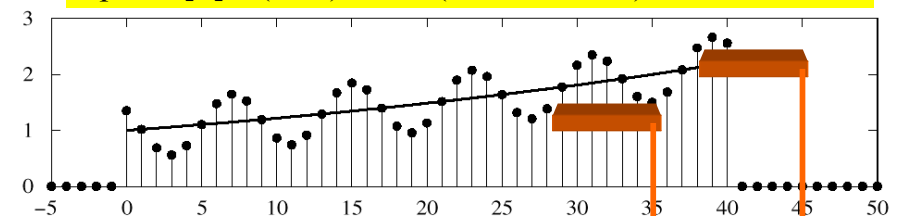
USE PAST VALUES

Output of 3-Point Running-Average Filter



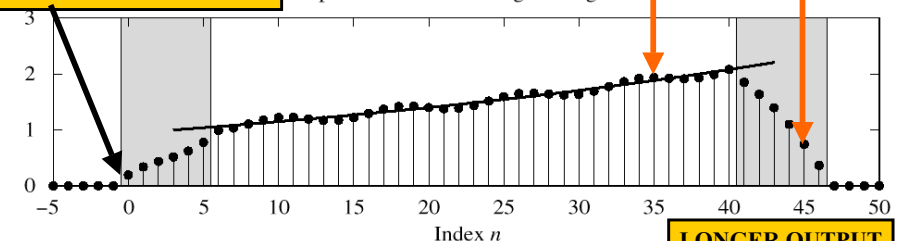
7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



CAUSAL: Use Previous

Output of 7-Point Running-Average Filter



LONGER OUTPUT