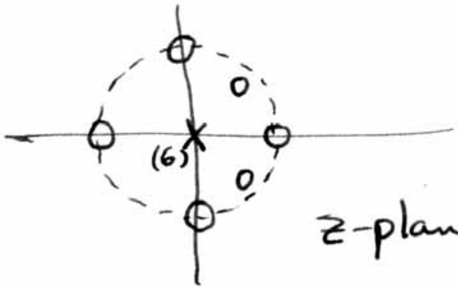
PROBLEM 7.16:

(a) $H(z)$ has 6 zeros & 6 poles at $z=0$

The zeros are: $z = \pm 1,\ \pm j,\ 0.8\, e^{\pm j\pi/4}$



z-plane

(b) $w[n] = x[n] - x[n-4]$

$$\Rightarrow \quad H_1(z) = 1 - z^{-4} = (1+z^{-2})(1-z^{-2}).$$

To get $H_2(z)$ divide:

$$H_2(z) = \frac{H(z)}{H_1(z)} = \left(1 - 0.8 e^{-j\pi/4} z^{-1}\right) \cdot \left(1 - 0.8 e^{+j\pi/4} z^{-1}\right).$$

$$H_2(z) = 1 - 1.6 \cos \tfrac{\pi}{4}\, z^{-1} + .64 z^{-2}$$

(c) $$y[n] = x[n] - \underbrace{(0.8\sqrt{2})}_{= 1.1314} x[n-1] + 0.64\, x[n-2].$$

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PROBLEM 5.3:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) MAKE A TABLE:

n	<0	0	1	2	3	4	5	6	7	≥ 8
$x[n]$	0	1	2	3	2	1	1	1	1	1
$y[n]$	0	2	1	2	-1	2	3	1	1	1 ......

$$y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2(1) = 2$$

$$y[1] = 2x[1] - 3x[0] + 2x[-1] = 2(2) - 3(1) = 1$$

$$y[2] = 2x[2] - 3x[1] + 2x[0] = 2(3) - 3(2) + 2(1) = 2$$

$$y[3] = 2(2) - 3(3) + 2(2) = -1$$

$$y[4] = 2(1) - 3(2) + 2(3) = 2$$

$$y[5] = 2(1) - 3(1) + 2(2) = 3$$

$$y[6] = 2(1) - 3(1) + 2(1) = 1$$

$$y[7] = 2(1) - 3(1) + 2(1) = 1$$

$$y[8] = 2(1) - 3(1) + 2(1) = 1$$

(c) Impulse Response

$$h[0] = 2(1) - 3(0) + 2(0) = 2$$

$$h[1] = 2(0) - 3(1) + 2(0) = -3$$

$$h[2] = 2(0) - 3(0) + 2(0) = 2$$

Notice $h[n]$ just "reads out" the filter coefficients:

i.e., $h[n] = b_n$

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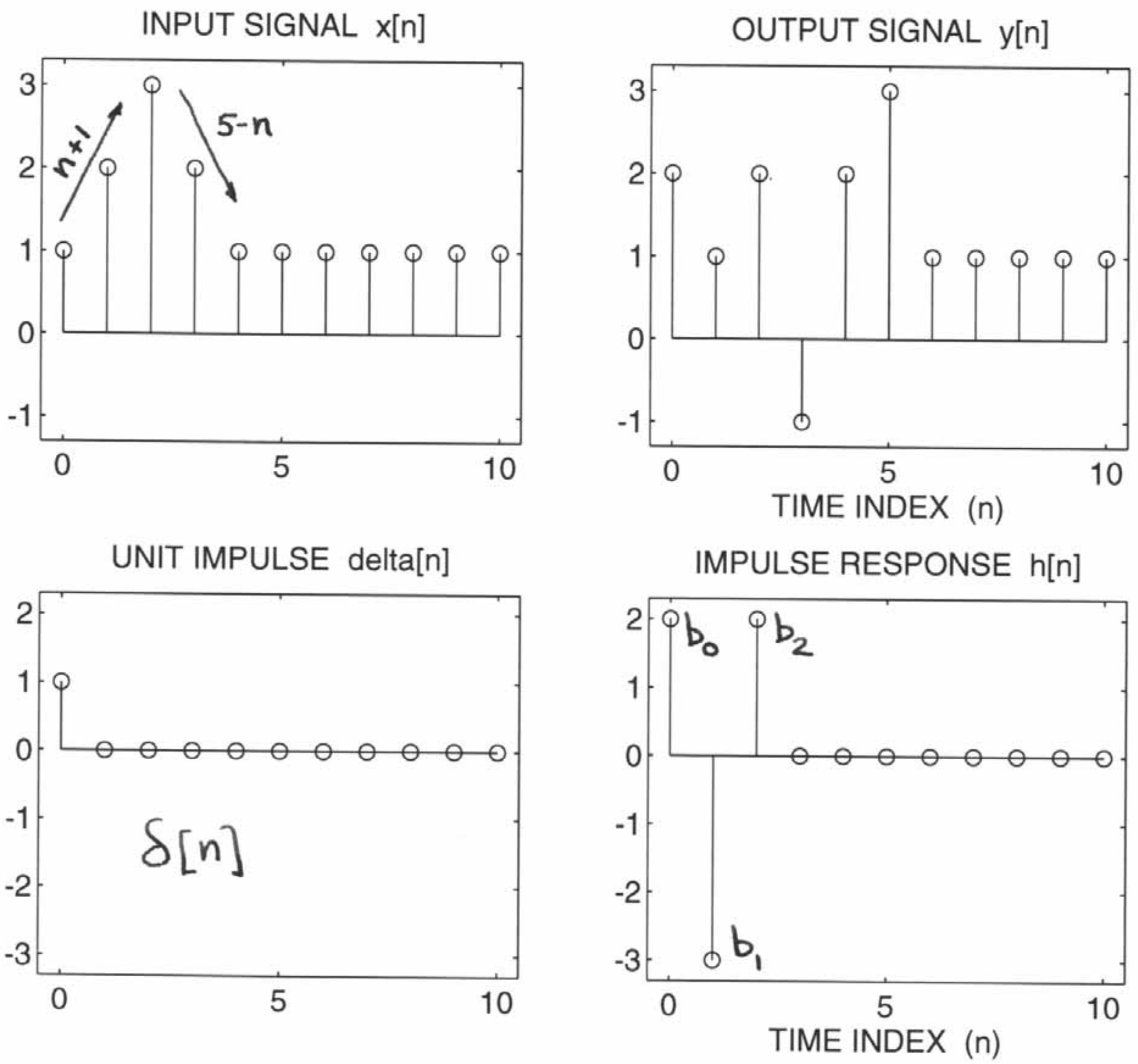
PROBLEM 5.3 (more):

Plots via MATLAB



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PROBLEM 5.6:

Plots for parts (a), (b) and (c) are below.

(d) This general solution will also apply to part (c).

$$x[n] = a^n u[n] \qquad y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} u[n-k]$$

There are 3 cases.

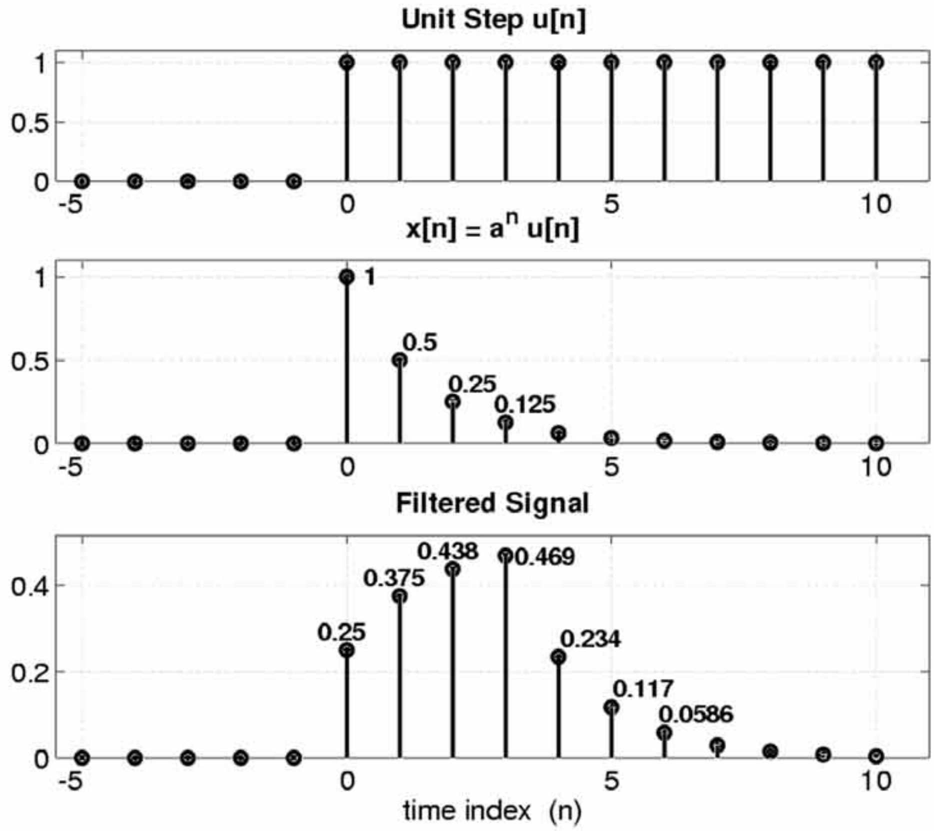
1. $n<0. \Rightarrow y[n]=0$ because $u[n-k]$ is always zero

2. $0 \le n \le L-1$ $\quad y[n] = \frac{1}{L}\sum_{k=0}^{n} a^{n-k} u[n-k] = \frac{a^n}{L}\sum_{k=0}^{n} a^{-k}$

$$y[n] = \frac{a^n}{L}\left(\frac{1-a^{-n-1}}{1-a^{-1}}\right) = \frac{1}{L}\left(\frac{a^{n+1}-1}{a-1}\right)$$

3. $n \ge L$ $\quad y[n] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} \cancel{u[n-k]}^{\,1} = \frac{a^n}{L}\sum_{k=0}^{L-1} a^{-k}$

$$= \frac{a^n}{L}\left(\frac{1-a^{-L}}{1-a^{-1}}\right) = \frac{a^n}{L}\left(\frac{a^L-1}{a^L-a^{L-1}}\right) \quad \text{for } n \ge L.$$



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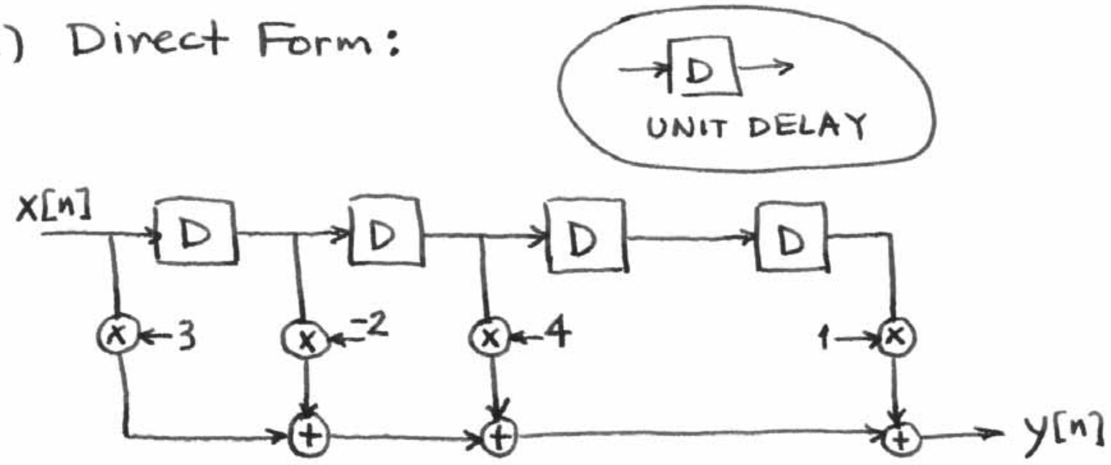
PROBLEM 5.11:

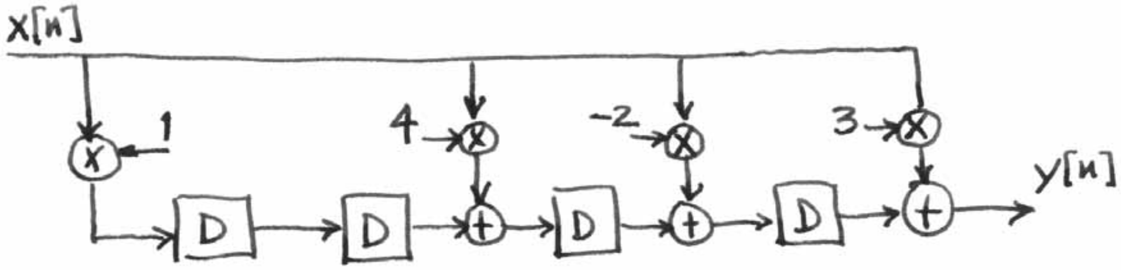
$$h[n] = 3\delta[n] - 2\delta[n-1] + 4\delta[n-2] + \delta[n-4]$$

$$\Rightarrow \quad y[n] = 3x[n] - 2x[n-1] + 4x[n-2] + x[n-4]$$

(a) Direct Form:



(b) Transposed Direct Form:



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PROBLEM 5.14:

(a) $h[n] = \delta[n-2] \Rightarrow$ filter is a delay by 2

$$y[n] = u[n-3] - u[n-6]$$

To find $x[n]$ we need to "un-delay" $y[n]$.

$$\Rightarrow \quad x[n] = u[n-1] - u[n-4]$$

(b) First-difference FIR $\Rightarrow h[n] = \delta[n] - \delta[n-1]$

The first-difference filter has a nonzero output at n when $x[n]$ & $x[n-1]$ are not equal.

If $y[n] = \delta[n] - \delta[n-4]$, then the input $x[n]$ changes value at $n=0$ and $n=4$. At $n=0$, it jumps up by one; at $n=4$, it jumps down.

$$\Rightarrow \quad x[n] = u[n] - u[n-4]$$

jump up by one

jump down

(c) 4-pt averager: $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

If $y[n] = -5\delta[n] - 5\delta[n-2]$

$$y[0] = -5 = \frac{1}{4}\left(x[0] + x[-1] + x[-2] + x[-3]\right)$$

** if we assume $x[n] = 0$ for $n<0$, then $x[0] = -20$

$$y[1] = 0 = \frac{1}{4}\left(x[1] + x[0] + x[-1] + x[-2]\right) = \frac{1}{4}x[1] - 5$$

$$\Rightarrow x[1] = 20$$

$$\left.\begin{aligned} y[2] = -5 &= \frac{1}{4}\left(x[2] + x[1] + x[0] + x[-1]\right) \\ &= \frac{1}{4}\left(x[2] + 20 - 20 + 0\right) = \frac{1}{4}x[2] \end{aligned}\right\} x[2] = -20$$

$$y[3] = 0 = \frac{1}{4}\left(x[3] + x[2] + x[1] + x[0]\right) \Rightarrow x[3] = -20$$

$$\Rightarrow \quad x[n] = \begin{cases} 0 & \text{for } n<0 \\ -20 & \text{for } n \text{ even} \\ 20 & \text{for } n \text{ odd} \end{cases}$$

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PROBLEM 5.17:

(a) $h_1[n] = \delta[n] - \delta[n-1]$

$h_2[n] = \delta[n] + \delta[n-2]$

$h_3[n] = \delta[n-1] + \delta[n-2]$

(b) The overall $h[n]$ is the convolution of the $h_i[n]$.

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$h_1[n] * h_2[n] = (\delta[n] - \delta[n-1]) * (\delta[n] + \delta[n-2])$$

$$= \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

Now convolve with $h_3[n]$

$$\begin{array}{cccccc}
1 & -1 & 1 & -1 & & \\
0 & 1 & 1 & & & \\
\hline
0 & 0 & 0 & 0 & & \\
 & 1 & -1 & 1 & -1 & \\
 & & 1 & -1 & 1 & -1 \\
\hline
0 & 1 & 0 & 0 & 0 & -1 \\
n=0 & \uparrow_{n=1} & & & & \uparrow_{n=5}
\end{array}$$

$$\boxed{h[n] = \delta[n-1] - \delta[n-5]}$$

(c) $y[n] = h[n] * x[n]$

$$= (\delta[n-1] - \delta[n-5]) * x[n]$$

$$y[n] = x[n-1] - x[n-5]$$

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PROBLEM 6.3:

$$y[n] = x[-n]$$

(a) $x[n] = Ae^{j\varphi}e^{+j\hat{\omega}n}$

$$y[n] = Ae^{j\varphi}e^{j\hat{\omega}(-n)} = Ae^{j\varphi}e^{-j\hat{\omega}n}$$

(b) NO.

The output cannot be written as

$$y[n] = \mathcal{H}(\hat{\omega})Ae^{j\varphi}e^{j\hat{\omega}n}$$

because the frequency has changed from $+\hat{\omega}$ to $-\hat{\omega}$.

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PROBLEM 6.8:

(a) $\mathcal{H}(\hat{\omega}) = (1+e^{-j\hat{\omega}})(1-\underbrace{2\cos(2\pi/3)}_{=2(1/2)=1}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$

$$= (1+e^{-j\hat{\omega}})(1-e^{-j\hat{\omega}}+e^{-j2\hat{\omega}})$$

$$= 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

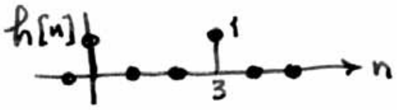
$$= 1 + e^{-j3\hat{\omega}}$$

Difference Equation:

$$y[n] = x[n] + x[n-3]$$

(b) when $x[n] = \delta[n]$, $y[n] = \delta[n] + \delta[n-3]$

This is $h[n]$, the impulse response



(c) Need to find where $\mathcal{H}(\hat{\omega}) = 0$.

$$1 + e^{-j3\hat{\omega}} = 0$$

$$e^{-j3\hat{\omega}} = -1 = e^{j\pi}e^{j2\pi\ell}$$

$$\Rightarrow e^{j\hat{\omega}} = e^{-j\pi/3}e^{-j2\pi\ell/3}$$

$$\Rightarrow \hat{\omega} = -\frac{\pi}{3} - \frac{2\pi}{3}\ell \qquad \ell = 0, 1, 2.$$

$\hat{\omega} = -\frac{\pi}{3}$, $-\pi$, and $-\frac{5\pi}{3}$ ← same as $+\pi/3$

$$y[n] = \mathcal{H}(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega}n}$$

Thus when $\mathcal{H}(\hat{\omega}) = 0$, the output is zero.

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PROBLEM 6.12:

$$y[n] = x[n] - 3x[n-1] + 3x[n-2] - x[n-3]$$

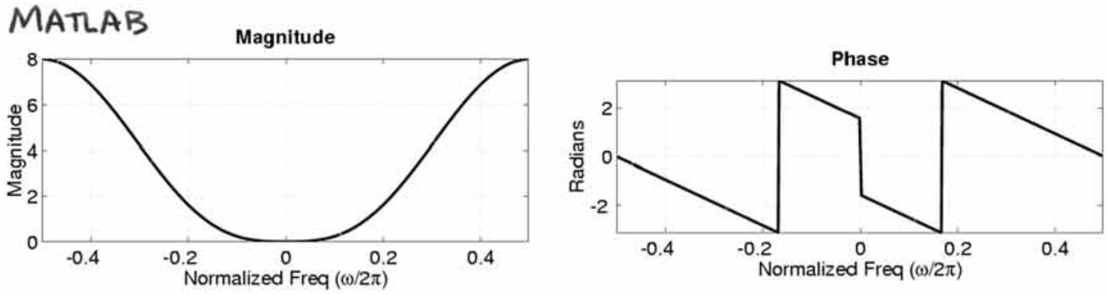
(a) use filter coeffs: $\{b_k\} = \{1, -3, 3, -1\}$

$$\mathcal{H}(\hat{\omega}) = 1 - 3e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} = (1 - e^{-j\hat{\omega}})^3$$

$$= e^{-j3\hat{\omega}/2}\left(\frac{e^{j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}{(2j)^3}\right)^3 (2j)^3 \qquad (2e^{j\pi/2})^3 = 8e^{-j\pi/2}$$

$$= 8e^{j(-\pi/2 - 3\hat{\omega}/2)} \sin^3(\hat{\omega}/2)$$

(b) MATLAB



(c) $x[n] = 10 + 4\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$

$$y[n] = 10\mathcal{H}(0) + 4\left|\mathcal{H}\left(\tfrac{\pi}{2}\right)\right| \cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4} + \angle\mathcal{H}\left(\tfrac{\pi}{2}\right)\right)$$

$$\mathcal{H}(\pi/2) = 8e^{-j(\pi/2 + 3\pi/4)} \sin^3\left(\tfrac{\pi}{4}\right)$$

$$= 8\left(\tfrac{\sqrt{2}}{2}\right)^3 e^{-j5\pi/4} = 2\sqrt{2}\, e^{-j5\pi/4}$$

$$\mathcal{H}(0) = 0$$

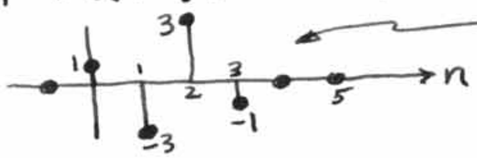
$$\Rightarrow \quad y[n] = 10(0) + 8\sqrt{2}\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4} - \tfrac{5\pi}{4}\right)$$

$$= 8\sqrt{2}\cos\left(\tfrac{\pi}{2}n - \pi\right)$$

(d) $x[n] = \delta[n]$

$$y[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$

h[n]



(e) Use superposition, so just add the results from (c) and (d)

$$y[n] = 8\sqrt{2}\cos\left(\tfrac{\pi}{2}n - \pi\right) + \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$

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PROBLEM 6.15:

$$x(t) = 10 + 8\cos(200\pi t) + 6\cos(500\pi t + \pi/4)$$

$$f_s = 1000$$

$$x[n] = x(t)\Big|_{t = n/f_s = n/1000}$$

$$= 10 + 8\cos\left(200\frac{\pi n}{1000}\right) + 6\cos\left(500\frac{\pi n}{1000} + \pi/4\right)$$

$$= 10 + 8\cos(0.2\pi n) + 6\cos(0.5\pi n + \pi/4)$$

Need $\mathcal{H}(0)$ Need $\mathcal{H}(0.2\pi)$ $\mathcal{H}(0.5\pi) = 0$

Use frequency response values from Prob. 6.14

$$\mathcal{H}(0) = 1 \qquad \mathcal{H}(0.2\pi) = 0.769\, e^{-j0.3\pi}$$

$$y[n] = 10 + (0.769)8\cos(0.2\pi n - 0.3\pi)$$

$$= 10 + 6.156\cos(0.2\pi n - 0.3\pi)$$

$$y(t) = y[n]\Big|_{n \leftarrow f_s t = 1000t}$$

$$= 10 + 6.156\cos(200\pi t - 0.3\pi)$$

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PROBLEM 6.20:

(a) The highest frequency in $x(t)$ is $\omega_0 = 2\pi(500)$
To avoid aliasing we must sample at $f_s > 2f_{MAX}$
$\Rightarrow f_s > 2(500\text{Hz}) = 1000$ samples/sec.

(b) $h[n] = \delta[n-10]$, f_s and ω_0 to be determined

$$x[n] = 10 + 20\cos(\omega_0 n/f_s + \pi/3)$$

$$y[n] = x[n-10] = 10 + 20\cos\left(\omega_0 \frac{(n-10)}{f_s} + \pi/3\right)$$

$$y(t) = y[n]\Big|_{n \leftarrow f_s t} = 10 + 20\cos\left(\frac{\omega_0}{f_s}(f_s t - 10) + \pi/3\right)$$

Since we want $y(t) = x(t-0.001)$, we need

$$\frac{\omega_0}{f_s}(10) = (0.001)\omega_0$$

$$\Rightarrow 10/f_s = 1/1000 \quad \Rightarrow f_s = 10{,}000\text{ Hz}$$

In order for the output frequency to be the same as the input frequency ω_0, there must be no aliasing. $\Rightarrow 2\omega_0 < 2\pi f_s$

$$\Rightarrow \omega_0 < 2\pi(500)\text{ rad/sec}$$

(c) To have $y(t)=A$, we need $y[n]=$constant.
Since $x[n] = 10 + 20\cos(\omega_0 n/f_s + \pi/3)$ $\quad f_s = 2000$
the filter must "null out" the cosine term
$\Rightarrow \frac{\omega_0}{f_s} = \hat{\omega}_{NULL}$ where $\hat{\omega}_{NULL}$ is one of the zeros of $\mathcal{H}(\hat{\omega})$

$$\mathcal{H}(\hat{\omega}) = 0 \text{ when } \hat{\omega} = 2\pi/5,\ 4\pi/5,\ -2\pi/5,\ -4\pi/5$$

$$\therefore \omega_0 = f_s\hat{\omega}_{NULL} = \{2\pi(400), 2\pi(800), -2\pi(400), -2\pi(800)\}$$

We must include all aliases:

$2\pi(400)$, $2\pi(2400)$, $2\pi(4400)$, ... $\quad 2\pi(400 + 2000\ell)$
$2\pi(800)$, $2\pi(2800)$, $2\pi(4800)$, ... $\quad 2\pi(800 + 2000\ell)$
$2\pi(-400)$, $2\pi(1600)$, $2\pi(3600)$, ... $\quad 2\pi(-400 + 2000\ell)$
$2\pi(-800)$, $2\pi(1200)$, $2\pi(3200)$, ... $\quad 2\pi(-800 + 2000\ell)$

PROBLEM 7.3:

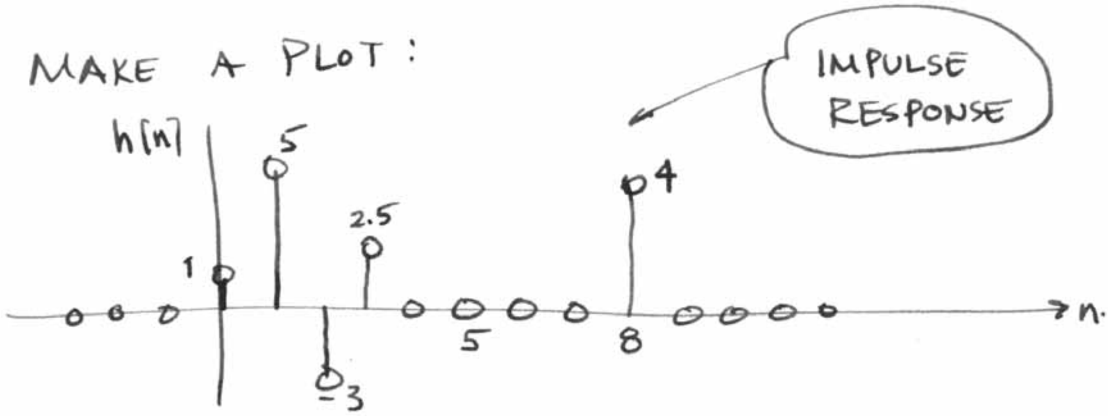
(a) $y[n] = x[n] + 5x[n-1] - 3x[n-2] + \frac{5}{2}x[n-3] + 4x[n-8]$

$$H(z) = 1 + 5z^{-1} - 3z^{-2} + \frac{5}{2}z^{-3} + 4z^{-8}$$

(b) when $x[n] = \delta[n]$, you can substitute.

$$h[n] = \delta[n] + 5\delta[n-1] - 3\delta[n-2] + \frac{5}{2}\delta[n-3] + 4\delta[n-8]$$

MAKE A PLOT:



NOTE:

The difference equation can be written as:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

Then the impulse response will just take on the values given by the $\{b_k\}$

$\therefore$ $h[0] = b_0$, $h[1] = b_1$, $h[2] = b_2$, ... etc.

PROBLEM 7.6:

(a) $Y_1(z) = H_1(z)X(z)$

$$Y(z) = H_2(z)Y_1(z) = H_2(z)\big(H_1(z)X(z)\big)$$
$$= \big(\underbrace{H_2(z)H_1(z)}_{H(z)}\big)X(z) \quad \text{because } H(z) = \frac{Y(z)}{X(z)}$$

(b) Since $H_2(z)H_1(z) = H_1(z)H_2(z)$ because $H_1(z)$ and $H_2(z)$ are scalar functions.

$$\Rightarrow Y(z) = H_1(z)\underbrace{H_2(z)X(z)}_{\text{means that } H_2(z) \text{ is applied first}}$$

(c) $H_1(z) = \frac{1}{3}(1+z^{-1}+z^{-2})$ by using the filter coeffs.

$$H(z) = H_2(z)H_1(z)$$
$$= \tfrac{1}{3}(1+z^{-1}+z^{-2})\cdot\tfrac{1}{3}(1+z^{-1}+z^{-2})$$
$$= \tfrac{1}{9}(1+2z^{-1}+3z^{-2}+2z^{-3}+z^{-4})$$

(d) Convert to difference equation (i.e., filter coeffs)

$$y[n] = \tfrac{1}{9}\big(x[n]+2x[n-1]+3x[n-2]+2x[n-3]+x[n-4]\big)$$

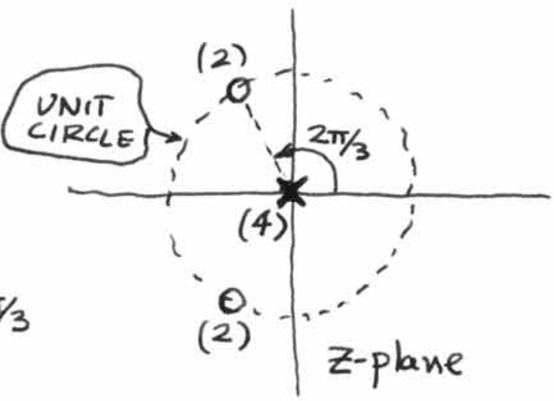
(e) Find the poles & zeros of $H_2(z)$, then "double" them because $H_1(z) = H_2(z)$.

$$H_2(z) = \tfrac{1}{3}z^{-2}(\underbrace{z^2+z+1})$$

$\frac{1}{z^2}$ contributes two poles at $z=0$

zeros are:

$$\frac{-1\pm\sqrt{1-4}}{2} = -\tfrac{1}{2}\pm j\tfrac{\sqrt{3}}{2} = e^{\pm j2\pi/3}$$



PROBLEM 7.6 (more):

(f)

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$
$$= \frac{1}{9}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)^2$$
$$= \frac{1}{9}e^{-j2\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)^2$$
$$= \frac{1}{9}e^{-j2\hat{\omega}}\left(1 + 2\cos(\hat{\omega})\right)^2$$

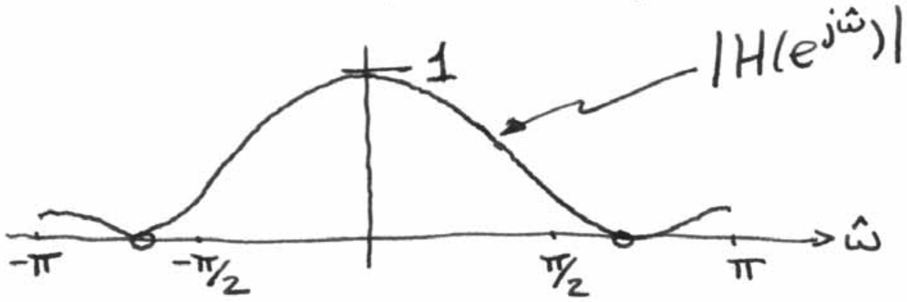
$$|H(e^{j\hat{\omega}})| = \frac{1}{9}\left(1 + 2\cos(\hat{\omega})\right)^2$$

At $\hat{\omega} = 0$, $|H| = \frac{1}{9}(3)^2 = 1$

At $\hat{\omega} = \pi/2$, $|H| = \frac{1}{9}(1)^2 = 1/9$

At $\hat{\omega} = 2\pi/3$, $|H| = 0$ because there is a zero on the unit circle.

At $\hat{\omega} = \pi$, $|H| = \frac{1}{9}(1-2)^2 = 1/9$



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PROBLEM 7.12:

(a)

MULTIPLY OUTER FACTORS

$$H(z) = (1-z^{-1})(1+z^{-2})(1+z^{-1})$$
$$= (1-z^{-2})(1+z^{-2}) = 1-z^{-4}$$

$\therefore \quad y[n] = x[n] - x[n-4]$

(b) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j4\hat{\omega}}$

(c) $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(e^{+j2\hat{\omega}} - e^{-j2\hat{\omega}})$
$$= 2j\, e^{-j2\hat{\omega}} \sin 2\hat{\omega} = (2\sin 2\hat{\omega})\, e^{j(\pi/2 - 2\hat{\omega})}.$$

MAG: $2\sin 2\hat{\omega}$

ALTHOUGH THIS HAS A SIGN CHANGE FOR $\hat{\omega} < 0$

PHASE: $\pi/2 - 2\hat{\omega}$

(d) BLOCK WHEN $H(e^{j\hat{\omega}}) = 0$

$\therefore$ SOLVE $2\sin 2\hat{\omega} = 0$

$\Rightarrow \hat{\omega} = 0, \pi/2, \pi, -\pi/2$

(e) Need $H(e^{j\pi/3})$ because that is the frequency of the input.

$$H(e^{j\pi/3}) = \left(2\sin\tfrac{2\pi}{3}\right) e^{j(\pi/2 - 2\pi/3)}$$
$$= 2\left(\tfrac{-\sqrt{3}}{2}\right) e^{j(3\pi/6 - 4\pi/6)}$$
$$= -\sqrt{3}e^{-j\pi/6} = \sqrt{3}e^{j\pi}e^{-j\pi/6} = \sqrt{3}\,e^{j5\pi/6}$$

$\therefore$ OUTPUT IS: $\quad y[n] = \sqrt{3}\cos\left(\frac{\pi n}{3} + \frac{5\pi}{6}\right)$

PROBLEM 7.18:

(a) $H_1(z) = H_2(z) = 1 + z^{-1} + z^{-2} + z^{-3}$

(b) $H(z) = H_1(z)H_2(z) = (1 + z^{-1} + z^{-2} + z^{-3})^2$

(c) Multiply out the product:

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}$$

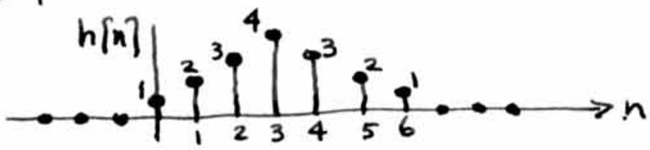
Invert term-by-term:

$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 3\delta[n-4] + 2\delta[n-5] + \delta[n-6]$$

(d) Use the polynomial coeffs of $H(z)$ as filter coefficients:

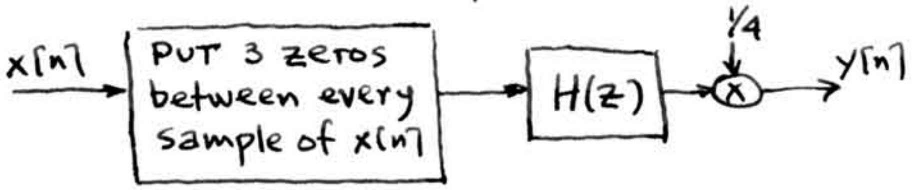
$$y[n] = x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 3x[n-4] + 2x[n-5] + x[n-6]$$

(e) Linear interpolation uses a triangularly shaped impulse response, which is exactly the form of $h[n]$.



There are <u>two</u> problems with using $h[n]$ directly: The value at the peak of the triangle is 4, not 1, so we need a scale factor of $\frac{1}{4}$. The peak value is at $n=3$, not $n=0$, so the output is time-shifted – delayed by 3.

Procedure: (to interpolate $x[n]$ by 4)



The output $y[n]$ is "$x[n]$ interpolated" but shifted by 3 samples.

PROBLEM 7.18 (more):

(f) Prove $H_1(z) = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

$$(1 - z^{-1}) H_1(z) = (1 - z^{-1})(1 + z^{-1} + z^{-2} + z^{-3})$$
$$= 1 + z^{-1} + z^{-2} + z^{-3} - z^{-1} - z^{-2} - z^{-3} - z^{-4}$$
$$= 1 - z^{-4}$$

(g) $H_1(z)$ and $H_2(z)$ have the same poles & zeros so we find the poles and zeros of $H_1(z)$ and plot them "twice" in the z-plane.

$$H_1(z) = \frac{1 - z^{-4}}{1 - z^{-1}} = \frac{z^4 - 1}{z^3(z-1)}$$

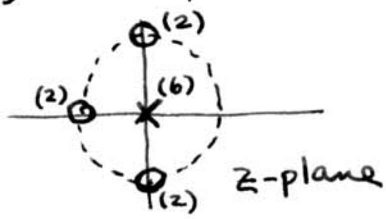
$z^4 - 1 \leftarrow$ ROOTS = 1, −1, j, −j

$z^3(z-1) \leftarrow$ ROOTS = 0, 1

The numerator and denominator both have roots at $z = 1$, so these cancel. We are left with 3 zeros at $z = -1, +j, -j$ and 3 poles at $z = 0$.

$\Rightarrow$ $H(z)$ has 6 zeros; two each at $z = -1, +j$ and $-j$

$H(z)$ has 6 poles at $z = 0$



(h) $H_1(e^{j\hat\omega}) = H_1(z)\big|_{z = e^{j\hat\omega}}$

$$= \frac{1 - e^{-j4\hat\omega}}{1 - e^{-j\hat\omega}} = \frac{e^{-j2\hat\omega}}{e^{-j\hat\omega/2}} \cdot \frac{(e^{j2\hat\omega} - e^{-j2\hat\omega})(\frac{1}{2j})}{(e^{j\hat\omega/2} - e^{-j\hat\omega/2})(1/2j)}$$

$$= e^{-j3\hat\omega/2} \frac{\sin(2\hat\omega)}{\sin(\frac{1}{2}\hat\omega)}$$

(i) $H(e^{j\hat\omega}) = H_1^2(e^{j\hat\omega}) \longrightarrow e^{-j3\hat\omega}\left(\dfrac{\sin(2\hat\omega)}{\sin(\frac{1}{2}\hat\omega)}\right)^2$