PROBLEM 2.4:

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \cdots$$

Separate the real and imaginary parts:

$$e^{j\theta} = \Big(\underbrace{1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots}_{\cos\theta}\Big) + j\Big(\underbrace{\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots}_{\sin\theta}\Big)$$

$$\therefore\ e^{j\theta} = \cos\theta + j\sin\theta$$

which proves Euler's formula.

PROBLEM 2.7:

(a) $3e^{j\pi/3} + 4e^{-j\pi/6} = \left(\frac{3}{2} + j\frac{3\sqrt{3}}{2}\right) + \left(4\frac{\sqrt{3}}{2} - j\frac{4}{2}\right)$

$= 4.9641 + j0.5981$

$= 5e^{j0.12}$ NOTE: $0.12 \text{ rad} = 6.87^\circ$

(b) $\sqrt{3} - j3 = \sqrt{3+3^2}\, e^{-j\pi/3} = \sqrt{12}\, e^{-j\pi/3}$.

$\Rightarrow \left(\sqrt{3} - j3\right)^{10} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{10}$

$= 2^{10} 3^5 e^{-j10\pi/3}$ $\quad -\frac{10\pi}{3} + 4\pi = \frac{-10\pi + 12\pi}{3} = 2\pi/3$

$= 248{,}832\, e^{+j2\pi/3} = -124{,}416 + j215{,}494.83$

(c) $\dfrac{1}{\sqrt{3} - j3} = \dfrac{1}{\sqrt{12}\, e^{-j\pi/3}} = \dfrac{1}{\sqrt{12}} e^{+j\pi/3} = 0.2887 e^{+j\pi/3}$

$= 0.14434 + j0.25$

(d) $\left(\sqrt{3} - j3\right)^{1/3} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{1/3} = \left(\sqrt{12}\, e^{-j(\pi/3 + 2\pi\ell)}\right)^{1/3}$

$= 12^{1/6} e^{-j(\pi/9 + \frac{2\pi}{3}\ell)}$ $\quad \ell = \text{integer}$

Need $\ell = 0, 1, 2$

There are 3 answers:

$1.513 e^{-j\pi/9} = 1.422 - j0.5175$

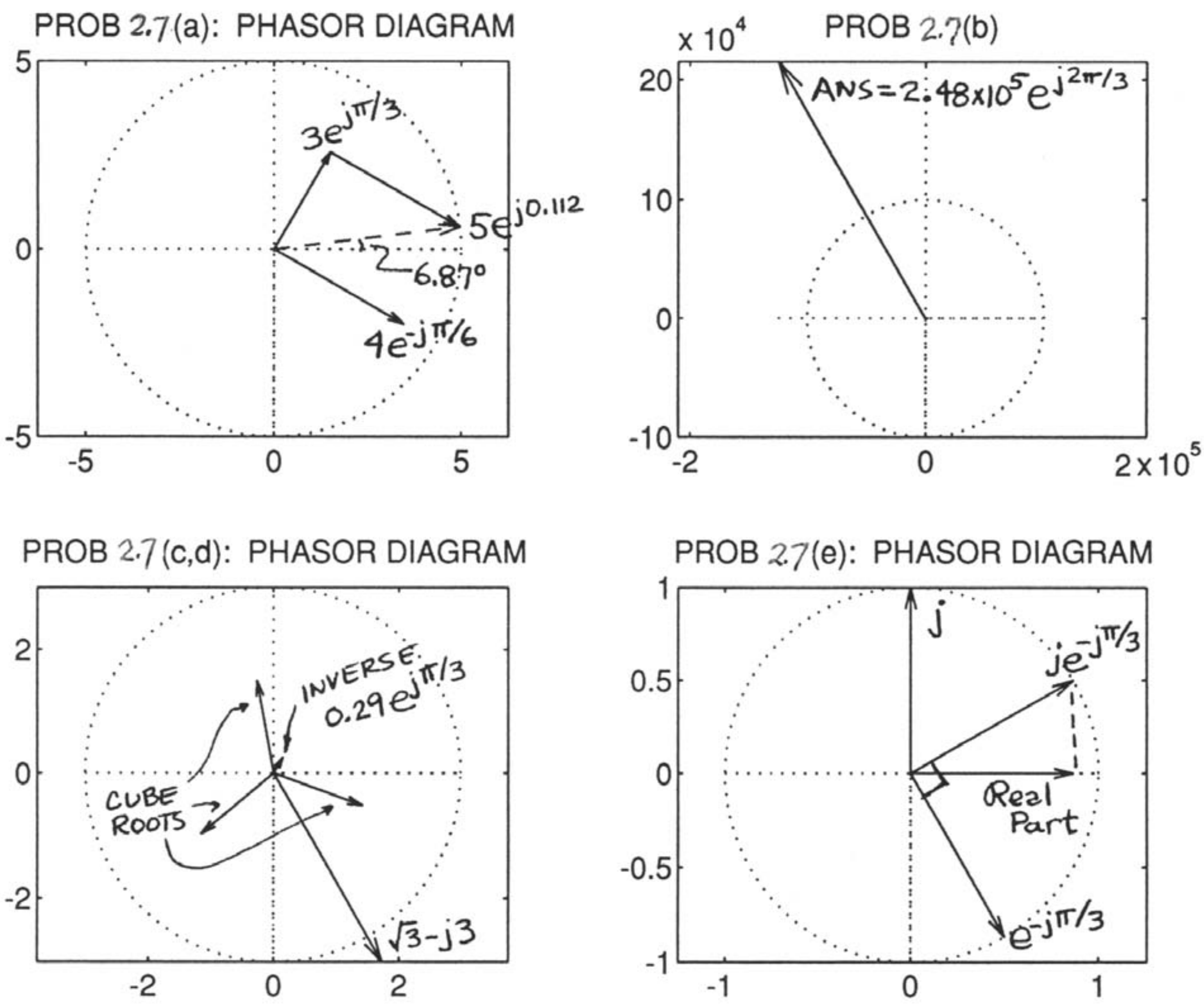
$1.513\, e^{-j7\pi/9} = -1.159 - j0.9726$

$1.513\, e^{-j13\pi/9} = 1.513 e^{+j5\pi/9} = -0.2627 + j1.49$

(e) $\Re e\{j e^{-j\pi/3}\} = \Re e\{e^{j\pi/2} e^{-j\pi/3}\}$.

$= \Re e\{e^{j\pi/6}\} = \cos(\pi/6) = \frac{\sqrt{3}}{2} = 0.866$

PROBLEM 2.7 (more):



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PROBLEM 2.15:

Express $x(t) = 5\cos(\omega t + \frac{1}{3}\pi) + 7\cos(\omega t - \frac{5}{4}\pi) + 3\cos(\omega t)$ in the form $x(t) = A\cos(\omega t + \phi)$.

Solution:
Convert to phasors:

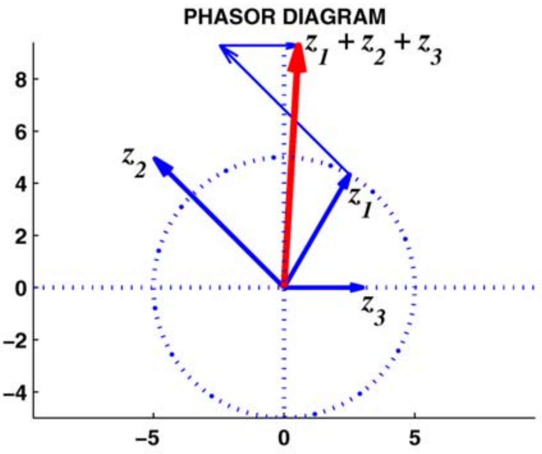
$$
\begin{aligned}
5\cos(\omega t + \tfrac{1}{3}\pi) &\longrightarrow z_1 = 5e^{j\pi/3} = 2.5 + j4.33 \\
7\cos(\omega t - \tfrac{5}{4}\pi) &\longrightarrow z_2 = 7e^{j5\pi/34} = -4.95 + j4.95 \\
3\cos(\omega t) &\longrightarrow z_3 = 3e^{j0} = 3 + j0
\end{aligned}
$$

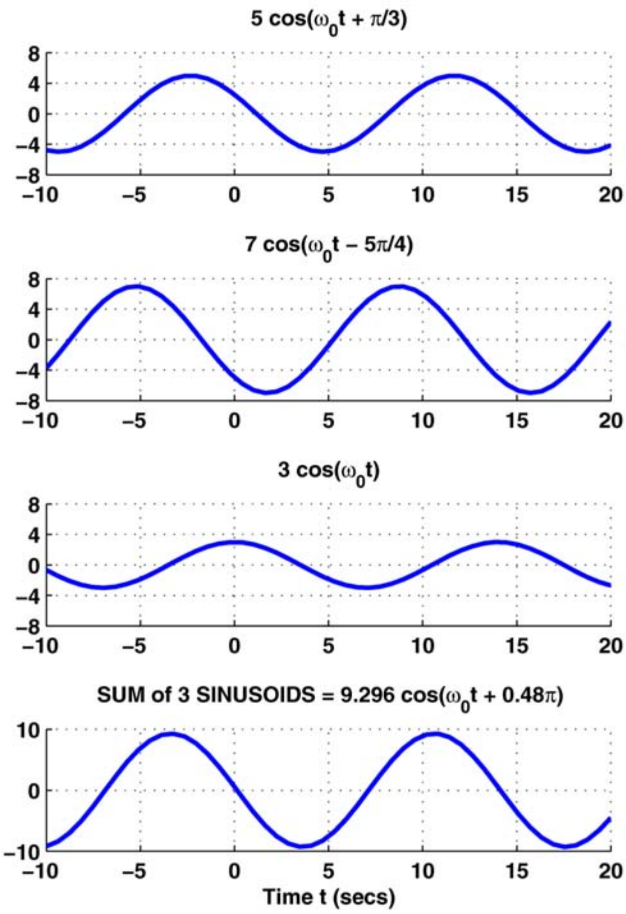
Perform the phasor addition to get:

$$z_1 + z_2 + z_3 = (2.5 + j4.33) + (-4.95 + j4.95) + (3) = 0.5503 + j9.28 = 9.296e^{j0.48\pi}$$

Thus, the resultant sinusoid is:

$$x(t) = 9.296\cos(\omega t + 0.48\pi)$$





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PROBLEM 2.20:

$$x[n] = 7e^{j(0.22\pi n - 0.25\pi)}$$

$$y[n] = 7e^{j(0.22\pi(n+1) - 0.25\pi)} - 14e^{j(0.22\pi n - 0.25\pi)} + 7e^{j(0.22\pi(n-1) - 0.25\pi)}$$

$$= 7e^{-j0.25\pi} e^{j0.22\pi n} \Big(\underbrace{e^{j0.22\pi} - 2 + e^{-j0.22\pi}}_{2\cos(0.22\pi) - 2 = -0.459 = 0.459e^{j\pi}} \Big)$$

$\therefore$

$$y[n] = 7(0.459e^{j\pi})e^{-j0.25\pi} e^{j0.22\pi n}$$

$$= 3.213\, e^{j0.75\pi} e^{j0.22\pi n}$$

$$\boxed{\begin{aligned} \hat{\omega}_0 &= 0.22\pi \\ A &= 3.213 \\ \varphi &= 0.75\pi \end{aligned}}$$

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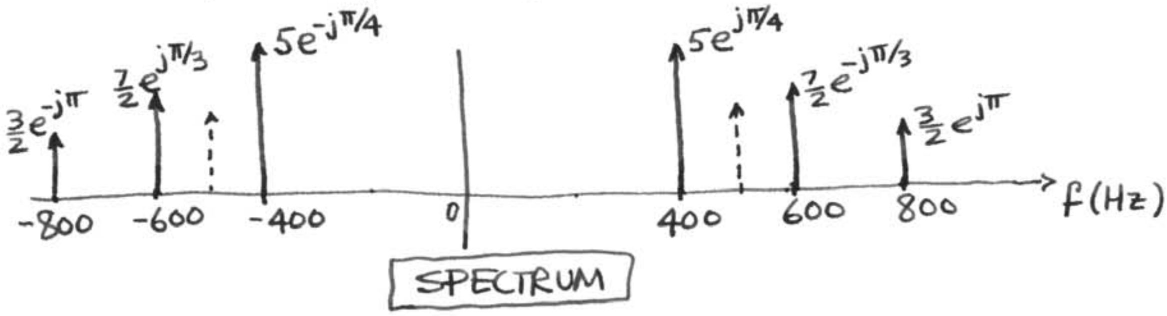
PROBLEM 3.1:

(a) There are 3 components

$$10\cos(800\pi t + \pi/4) = \Re e\{10e^{j\pi/4}e^{j800\pi t}\} \qquad \text{freq} = 400\text{ Hz}$$

$$7\cos(1200\pi t - \pi/3) = \Re e\{7e^{-j\pi/3}e^{j1200\pi t}\} \qquad \text{freq} = 600\text{ Hz}$$

$$-3\cos(1600\pi t) = \Re e\{3e^{j\pi}e^{j1600\pi t}\} \qquad \text{freq} = 800\text{ Hz}$$



(b) $x(t)$ is periodic because there is a fundamental frequency $f = 200$ Hz that divides all 3 freqs. The period is the fundamental period $= 1/200$ sec

(c)

$$5\cos(1000\pi t + \pi/2) = \Re e\{5e^{j\pi/2}e^{j1000\pi t}\} \qquad \text{freq} = 500\text{ Hz}$$

The spectrum will have two additional lines at $f = -500$ Hz and $f = 500$ Hz

$\frac{5}{2}e^{-j\pi/2}$

component is $\frac{5}{2}e^{j\pi/2}$

The dotted lines in the sketch above show where these two lines will be.

Yes, $y(t)$ is periodic. The fundamental frequency is now $f_0 = 100$ Hz because it has to divide into 500 Hz as well as 400, 600 and 800.

The period is now $\frac{1}{100}$ sec.

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PROBLEM 3.4:

(a) $x(t) = \Re e\{A e^{j2\pi(f_c - f_\Delta)t}\} + \Re e\{B e^{j2\pi(f_c + f_\Delta)t}\}$

$$= \Re e\Big\{\underbrace{\left(A e^{-j2\pi f_\Delta t} + B e^{j2\pi f_\Delta t}\right)}_{\tilde{x}(t)} e^{j2\pi f_c t}\Big\}$$

(b) $\tilde{x}(t) = \Big((B+A)\cos(2\pi f_\Delta t) + j(B-A)\sin(2\pi f_\Delta t)\Big) e^{j2\pi f_c t}$

$$\Re e\{\tilde{x}(t)\} = (B+A)\cos(2\pi f_\Delta t)\cos(2\pi f_c t) - (B-A)\sin(2\pi f_\Delta t)\sin(2\pi f_c t)$$

$$\Longrightarrow C = B + A$$

$$D = -(B-A) = A - B$$

when $A = B = 1$, we get $C = 2$ and $D = 0$ so the sine terms drop out.

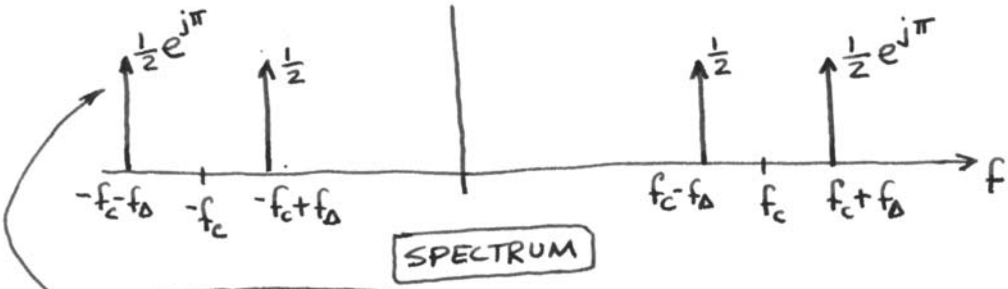
(c) Want $D = 2$ and $C = 0$

$$\left.\begin{array}{l} B + A = 0 \\ A - B = 2 \end{array}\right\} \Longrightarrow A = 1 \text{ and } B = -1$$

Return to the original expression for $x(t)$

$$x(t) = \tfrac{1}{2} e^{j2\pi(f_c - f_\Delta)t} + \tfrac{1}{2} e^{-j2\pi(f_c - f_\Delta)t} - \tfrac{1}{2} e^{j2\pi(f_c + f_\Delta)t} - \tfrac{1}{2} e^{-j2\pi(f_c + f_\Delta)t}$$

There are 4 terms



NOTE: $-\frac{1}{2} = \frac{1}{2} e^{j\pi}$

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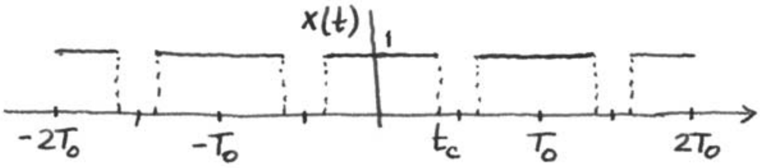
PROBLEM 3.9:

(a)



(b) $$X_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt = \frac{1}{T_0}\int_{-t_c}^{t_c} 1\,dt = \frac{2t_c}{T_0}$$

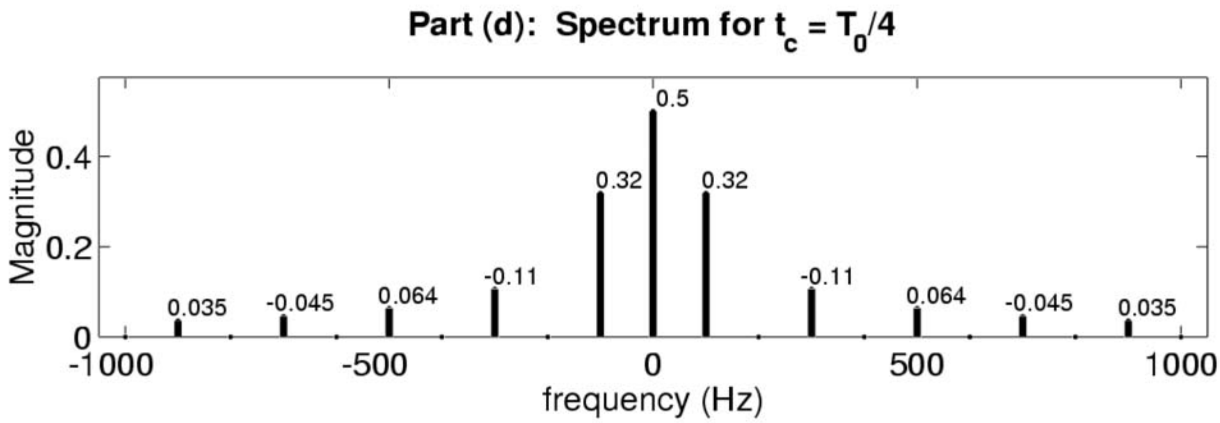
(c) $$X_k = \frac{2}{T_0}\int_{-t_c}^{t_c} 1\, e^{-jk\omega_0 t}\,dt = \frac{2}{T_0}\,\frac{e^{-jk\omega_0 t}}{(-jk\omega_0)}\Bigg|_{-t_c}^{t_c}$$

$$= \frac{2}{T_0}\,\frac{e^{-jk\omega_0 t_c} - e^{jk\omega_0 t_c}}{(-jk\omega_0)} = \frac{4}{k\omega_0 T_0}\sin(k\omega_0 t_c)$$

(d) $t_c = T_0/4$

In this case, the "sine" term is $\sin\left(\frac{2\pi k}{T_0}\cdot\frac{T_0}{4}\right) = \sin\left(\frac{\pi k}{2}\right)$

Therefore, $X_k = 0$ for k even and $k \neq 0$.



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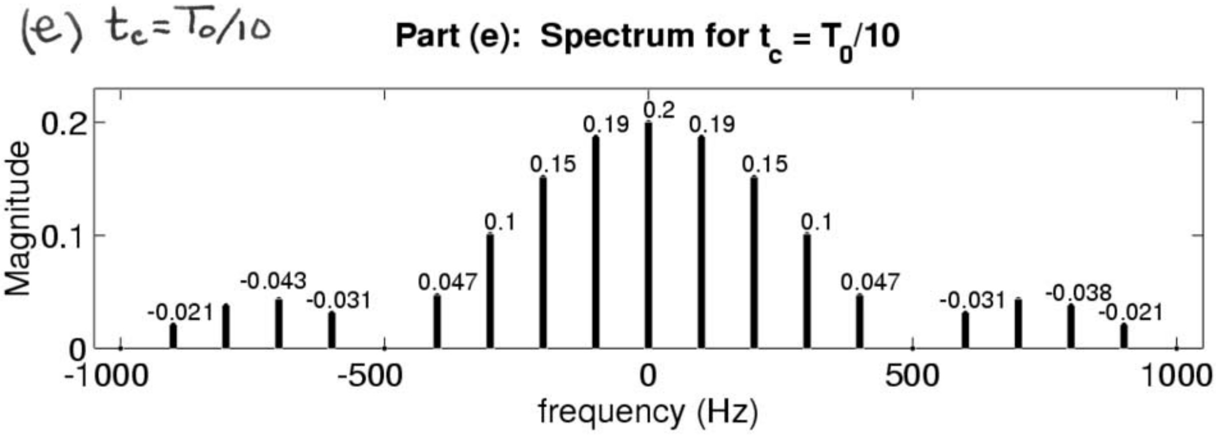
PROBLEM 3.9 (more):

(e) $t_c = T_0/10$



(f) When $t_c = T_0/10$ the high frequency components (above 500 Hz) are relatively larger, and there are more high freq. components. For example,

$$\left|\frac{X_7}{X_0}\right| = \frac{0.045}{0.5} = 0.09 \text{ for } t_c = T_0/4, \text{ but is } 0.215 \text{ for } t_c = T_0/10$$

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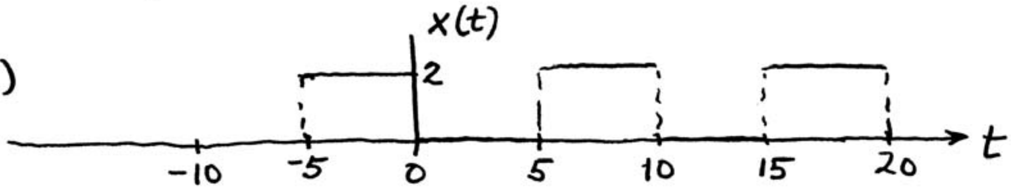
PROBLEM 3.12:

$$x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10 \end{cases} \qquad T_0 = 10 \text{ secs.}$$

(a)



(b) $a_0 = \frac{1}{T_0} \times \text{Area} = \frac{1}{10} \times (5 \times 2) = 1$

(c) $$a_1 = \frac{1}{10} \int_5^{10} 2 e^{-j(2\pi/10)t} \, dt$$

$$= \frac{1}{5} \left. \frac{e^{-j\pi t/5}}{(-j\pi/5)} \right|_5^{10} = \frac{e^{-j2\pi} - e^{-j\pi}}{-j\pi} = \frac{1-(-1)}{-j\pi} = \frac{2j}{\pi}$$

(d) $$y(t) = 1 + x(t) = 1 + \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$$

$$= (1 + a_0) + \sum_{k \ne 0} a_k e^{j\omega_0 k t}$$

$$\Rightarrow b_0 = 1 + a_0 \quad \text{and} \quad b_1 = a_1$$

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PROBLEM 3.19:

Characterize each time signal:

(a) 6 periods from $t=-2$ to $t=+3$ $\Rightarrow T = \frac{5}{6}$ sec
DC value $= 2$ $\qquad t_m > 0 \Rightarrow \varphi < 0$

(b) 3 periods from $t=0$ to $t=2$ $\Rightarrow T = \frac{2}{3}$ sec
DC value $= 0$ $\qquad \varphi = \pi$

(c) 6 periods from $t=-3$ to $t=2$ $\Rightarrow T = \frac{5}{6}$ sec
DC value $= 2$ $\qquad t_m < 0 \Rightarrow \varphi > 0$

(d) Period $\approx 3\frac{1}{3} = \frac{10}{3}$ secs. Two frequencies
DC value $= 0$

(e) 2 periods from $t=-2$ to $t=3$ $\Rightarrow T = 2.5$ secs
Two frequencies. $\qquad$ DC value $= 0$

(1) $\omega_0 = 2\pi(1.2) \Rightarrow T = 1/1.2 = 5/6$ sec
$\varphi = 0.5\pi > 0$ $\qquad$ DC $= 2$

(2) $\omega_0 = 2\pi(0.3)$ because 0.3 divides 0.6 & 1.5
$\Rightarrow T = 1/0.3 = 10/3$ secs. $\qquad$ DC $= 0$

(3) Like (1). $T = 5/6$ sec. $\qquad$ DC $= 2$
But $\varphi = -0.25\pi < 0$

(4) $\omega_0 = 2\pi(0.4) \Rightarrow T = 1/0.4 = 2.5$ secs
DC $= 0$

(5) $\varphi = \pi$ $\qquad T = 1/1.5 = \frac{2}{3}$ sec

Thus, the match is

(a) $\leftrightarrow$ (3) $\qquad$ (c) $\leftrightarrow$ (1) $\qquad$ (e) $\leftrightarrow$ (4)

(b) $\leftrightarrow$ (5) $\qquad$ (d) $\leftrightarrow$ (2)

PROBLEM 4.2:

$$x(t) = 7\sin(11\pi t) = 7\cos(11\pi t - \pi/2) \longrightarrow \boxed{A/D}_{f_s} \longrightarrow x[n] = A\cos(\hat{\omega}_0 n + \varphi).$$

(a) $f_s = 10$ samples/sec.

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{f_s}\right) = 7\cos\left(\frac{11\pi n}{10} - \pi/2\right).$$

$$= 7\cos\left(\frac{11\pi n}{10} - 2\pi n - \pi/2\right).$$

$$= 7\cos\left(-\frac{9\pi n}{10} - \pi/2\right) = 7\cos(.9\pi n + \pi/2).$$

$$\boxed{A = 7,\ \hat{\omega}_0 = 0.9\pi,\ \varphi = \pi/2}$$

(b) $f_s = 5$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{5}\right) = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

$$= 7\cos\left(\frac{\pi n}{5} - \frac{\pi}{2}\right)$$

$$\boxed{A = 7,\ \hat{\omega}_0 = \frac{\pi}{5},\ \varphi = -\frac{\pi}{2}}$$

(c) $f_s = 15$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{15}\right) = 7\cos\left(\frac{11\pi n}{15} - \frac{\pi}{2}\right)$$

$$A = 7,\ \hat{\omega}_0 = \frac{11\pi}{15} = 2\pi\left(\frac{5.5}{15}\right)\ \&\ \varphi = -\pi/2$$

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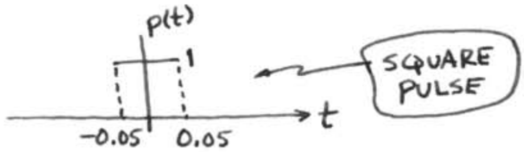
PROBLEM 4.6:

(a) $p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$

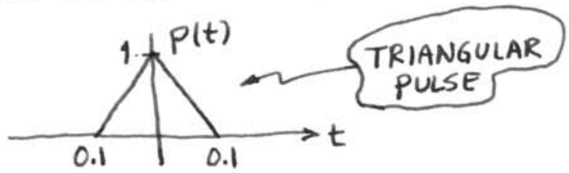


In the formula for $y(t)$

$$y(t) = \cdots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \cdots$$

The square pulses will not overlap, so the values of $y[n]$ will be extended over an interval of T_s.

(b) $p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$

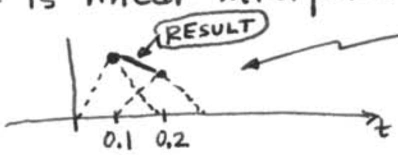


In this case, the neighboring terms do overlap

$$y(t) = \cdots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \cdots$$

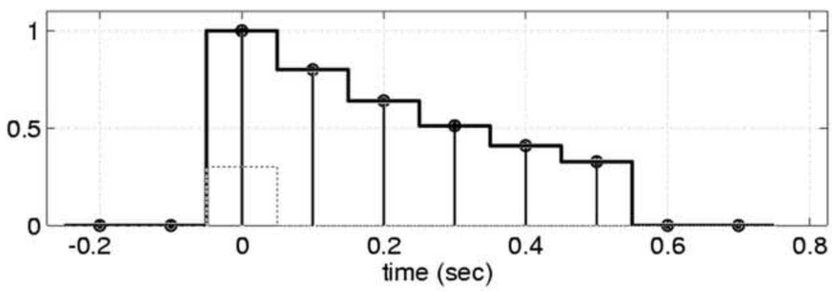
The result is linear interpolation.

Example:

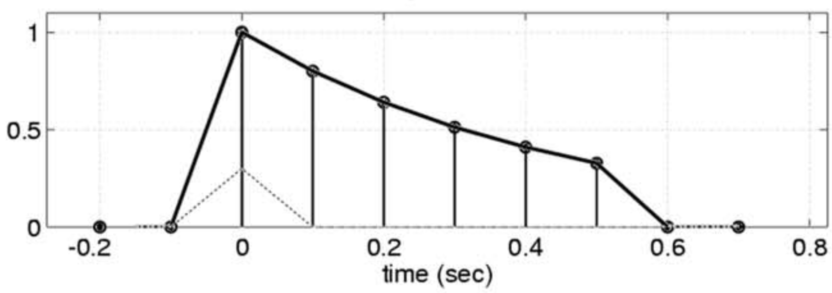


When we add these two triangles, the result between $t=0.1$ and $t=0.2$ is a straight line.

Problem 4.8(a) Square Pulse Shape



Problem 4.8(b) Triangular Reconstruction Pulse



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PROBLEM 4.11:

$$x(t) = [3 + \sin(\pi t)]\cos(13\pi t + \pi/2)$$

$\cos(\pi t - \pi/2)$

(a) Use *phasors* to show that $x(t)$ can be expressed in the form:

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3)$$

where $\omega_1 < \omega_2 < \omega_3$; i.e., find $A_1,\ A_2,\ A_3,\ \phi_1,\ \phi_2,\ \phi_3,\ \omega_1,\ \omega_2,\ \omega_3$ in terms of A, ω_0, and ω_c.

$$x(t) = \left[3 + \tfrac{1}{2}e^{j(\pi t - \pi/2)} + \tfrac{1}{2}e^{-j(\pi t - \pi/2)}\right]\left(\tfrac{1}{2}e^{j(13\pi t + \pi/2)} + \tfrac{1}{2}e^{-j(13\pi t + \pi/2)}\right)$$

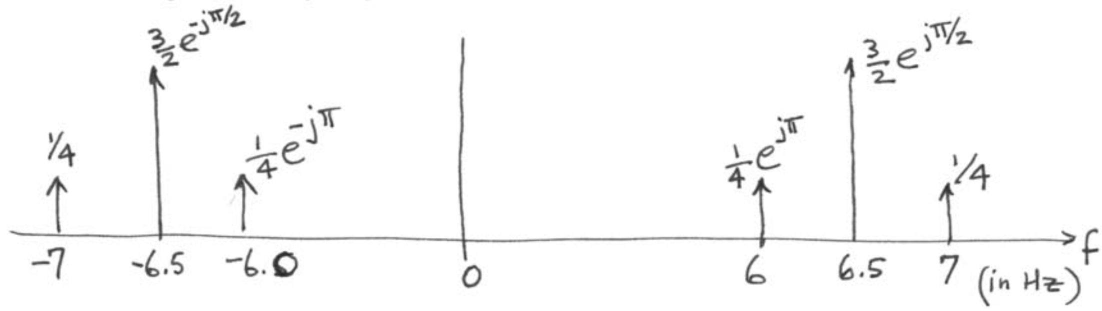
$$= \tfrac{3}{2}e^{j\pi/2}e^{j13\pi t} + \tfrac{3}{2}e^{-j\pi/2}e^{-j13\pi t} + \tfrac{1}{4}e^{j14\pi t} + \tfrac{1}{4}e^{-j14\pi t}$$

$$+ \tfrac{1}{4}e^{j\pi}e^{j12\pi t} + \tfrac{1}{4}e^{-j\pi}e^{-j12\pi t}$$

$$\Rightarrow 3\cos(13\pi t + \pi/2) + \tfrac{1}{2}\cos 14\pi t + \tfrac{1}{2}\cos(12\pi t + \pi)$$

$\omega_1 = 12\pi$	$A_1 = 1/2$	$\varphi_1 = \pi$
$\omega_2 = 13\pi$	$A_2 = 3$	$\varphi_2 = \pi/2$
$\omega_3 = 14\pi$	$A_3 = 1/2$	$\varphi_3 = 0$

(b) Sketch the two-sided spectrum of this signal on a frequency axis. Be sure to label important features of the plot. Label your plot in terms of the numerical values of the A_i's ϕ_i's and ω_i's.



(c) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

HIGHEST FREQ = 7 Hz.

$\Rightarrow F_{SAMP} \geq 2(7) = 14$ Hz

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PROBLEM 4.13:

Assume that the sampling rates of a C-to-D and D-to-C conversion system are equal, and the input to the Ideal C-to-D converter is

$$x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

(a) If the output of the ideal D-to-C Converter is equal to the input $x(t)$, i.e.,

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

what general statement can you make about the sampling frequency f_s in this case?

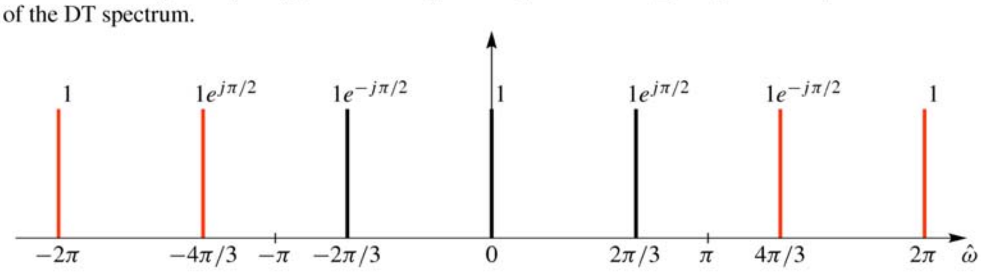
Solution: The sampling frequency must be greater than twice the highest frequency, because there was no aliasing. Thus, we can say that

$$F_s > 2 \times 150 = 300 \text{ Hz}$$

(b) If the sampling rate is $f_s = 250$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.* *Solution:* Replace t with $n/f_s = n/250$ to get

$$\begin{aligned} x[n] = x(n/250) &= 2\cos(2\pi(50)(n/250) + \pi/2) + \cos(2\pi(150)(n/250)) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.6)n) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.4)n) \end{aligned}$$

(c) Plot the spectrum of the signal in part (b) over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. The plot below shows the periodicity of the DT spectrum.



(d) If the output of the Ideal D-to-C Converter is

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + 1$$

determine the value of the sampling frequency f_s. (Remember that the input signal is $x(t)$ defined above.)

Solution: Since the frequency of 50 Hz is preserved, the other frequency of 150 Hz must have been aliased to 0 Hz. This can happen if the sampling frequency is $f_s = 150$ Hz, in which case the discrete-time signal is

$$\begin{aligned} x[n] = x(n/150) &= 2\cos(2\pi(50)(n/150) + \pi/2) + \cos(2\pi(150)(n/150)) \\ &= 2\cos(2\pi n/3 + \pi/2) + \cos(2\pi n) \\ &= 2\cos(2\pi n/3 + \pi/2) + 1 \end{aligned}$$

When $x[n]$ is reconstructed by the D/A converter running at $f_s = 150$ Hz, the final output will be

$$y(t) = x[n]\big|_{n \to f_s t} = 2\cos(2\pi(150t)/3 + \pi/2) + 1 = 2\cos(2\pi(50)t + \pi/2) + 1$$

PROBLEM 4.17:

(a) $\theta[n] = \pi(0.7 \times 10^{-3})n^2$ ← This is $\theta[n]$ in $\Re e\{e^{j\theta[n]}\}$

For $n=10$:

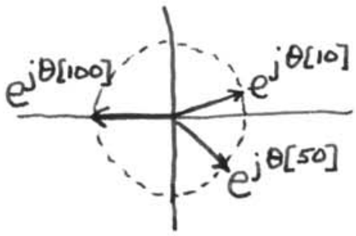
$$\theta[10] = \pi(0.7 \times 10^{-3})\,10^2 = 0.07\pi = 12.6^\circ$$

For $n=50$:

$$\theta[50] = \pi(0.7 \times 10^{-3})(50)^2 = \pi(0.7 \times 10^{-3} \times 25 \times 10^2) = \underset{315^\circ}{1.75\pi}$$

For $n=100$:

$$\theta[100] = \pi(0.7 \times 10^{-3})10^4 = 7\pi = \pi \text{ rads, or } 180^\circ$$



(c) Work part (c) before part (b)

$v[n] = \cos(0.7\pi n)$ $\qquad f_s = 8000\,\text{Hz}$

Ideal D/A ⇒ replace n with $f_s t$

$$v(t) = v[n]\Big|_{n=8000t} = \cos(0.7\pi \times 8000t) = \cos(2\pi(2800)t)$$

Freq is 2800 Hz

(b) $x[n] = \cos(\pi(0.7 \times 10^{-3})n^2)$ Replace n with $8000t$

$$x(t) = \cos(\pi(0.7 \times 10^{-3}) \times 64 \times 10^6 t^2)$$
$$= \cos(\pi(44.8 \times 10^3)t^2)$$

$\underbrace{n = 0, 1, \dots 200}$

$0 \le t \le \frac{200}{f_s} = 0.025\,\text{sec.}$

$$\psi(t) = \pi(44.8 \times 10^3)t^2$$

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t)$$
$$= \frac{1}{2\pi}(2\pi)(44.8 \times 10^3)t$$
$$= 44800\,t \quad \text{Hz}$$