

Signal Processing First

Lecture 24 Amplitude Modulation (AM)

8/22/2003

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LECTURE OBJECTIVES

- Review of FT properties
 - Convolution \leftrightarrow multiplication
 - Frequency shifting
- Sinewave Amplitude Modulation
 - AM radio
- Frequency-division multiplexing
 - FDM
- Reading: Chapter 12, Section 12-2

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Table of Easy FT Properties

Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j\left(\frac{\omega}{a}\right))$$

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Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

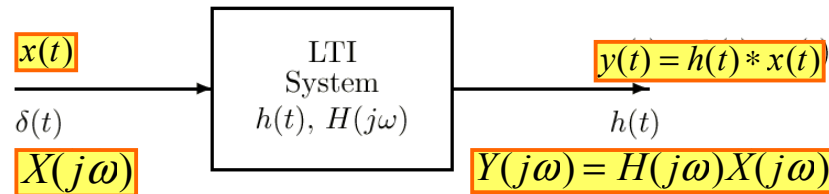
$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

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Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

corresponds to **MULTIPLICATION** in the frequency-domain

$$Y(j\omega) = H(j\omega)X(j\omega)$$

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Cosine Input to LTI System

$$Y(j\omega) = H(j\omega)X(j\omega)$$

$$= H(j\omega)[\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)]$$

$$= H(j\omega_0)\pi\delta(\omega - \omega_0) + H(-j\omega_0)\pi\delta(\omega + \omega_0)$$

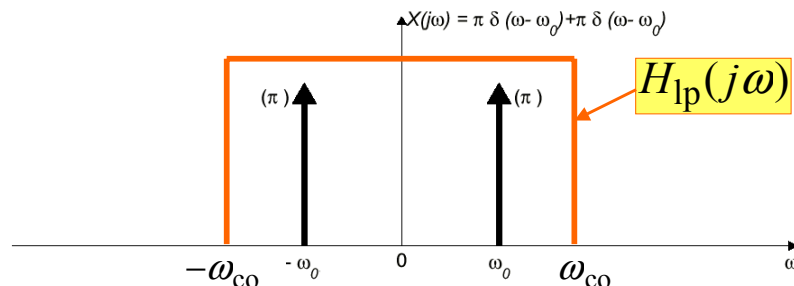
$$\begin{aligned} y(t) &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\ &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\ &= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0)) \end{aligned}$$

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Ideal Lowpass Filter



$$y(t) = x(t) \quad \text{if } \omega_0 < \omega_{co}$$

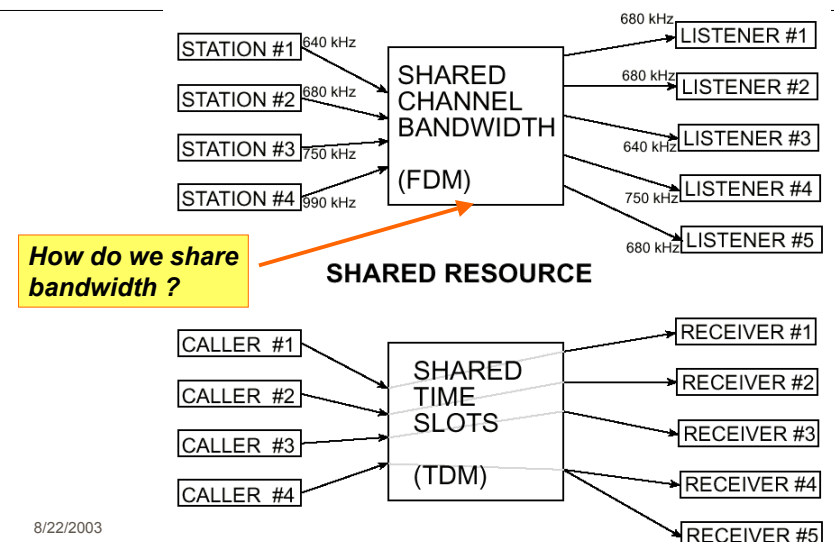
$$y(t) = 0 \quad \text{if } \omega_0 > \omega_{co}$$

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The way communication systems work



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Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

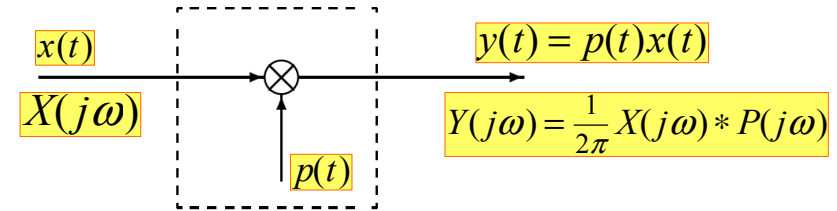
$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

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Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)P(j(\omega - \theta))d\theta$$

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$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases} \Leftrightarrow X(j\omega) = \frac{\sin(\omega T/2)}{(\omega/2)}$$

$$p(t) = \cos(\omega_0 t) \Leftrightarrow P(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$y(t) = x(t)\cos(\omega_0 t) \Leftrightarrow$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)]$$

$$Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_0)) + \frac{1}{2} X(j(\omega + \omega_0))$$

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Frequency Shifting Property

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt &= \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt \\ &= X(j(\omega - \omega_0)) \end{aligned}$$

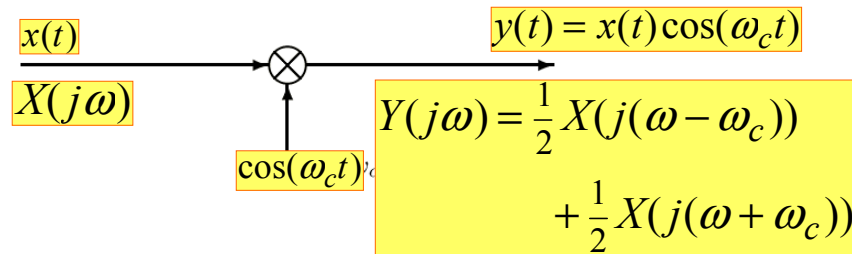
$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & \text{elsewhere} \end{cases}$$

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Amplitude Modulator



- $x(t)$ **modulates** the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of $X(j\omega)$.

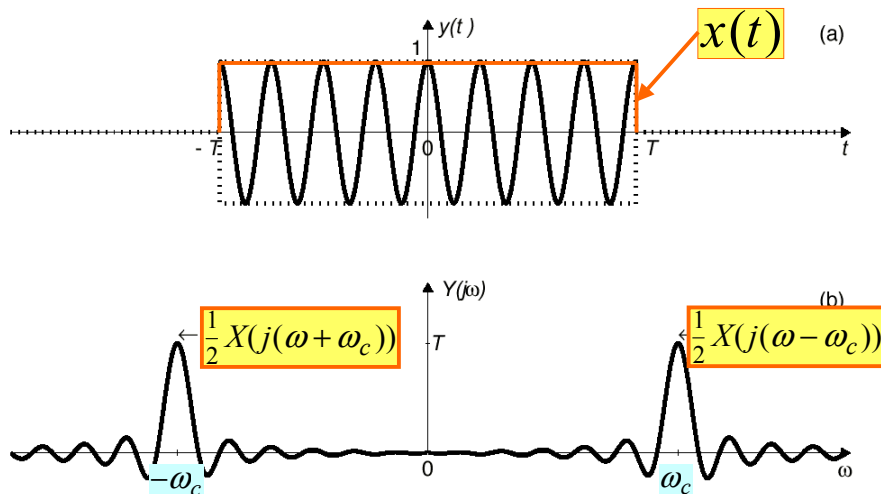
$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2 \frac{\sin(\omega T)}{(\omega)}$$

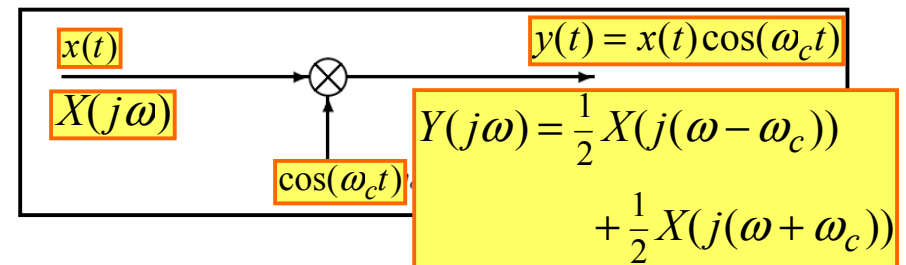
$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{\sin((\omega - \omega_c))}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c))}{(\omega + \omega_c)}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$



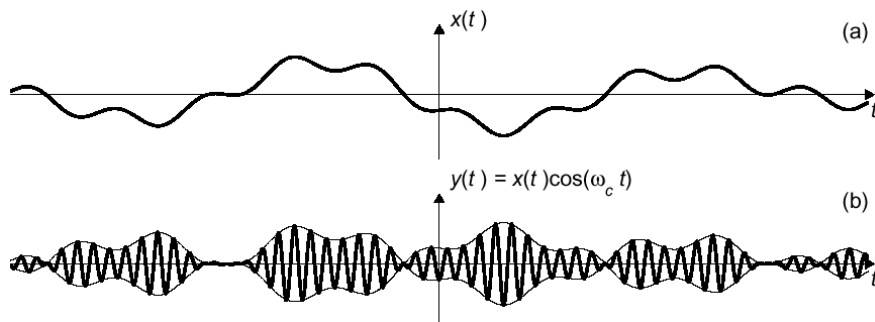
DSBAM Modulator



- If $X(j\omega) = 0$ for $|\omega| > \omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled **exact copies** of $X(j\omega)$.

DSBAM Waveform

- In the time-domain, the “envelope” of sine-wave peaks follows $|x(t)|$



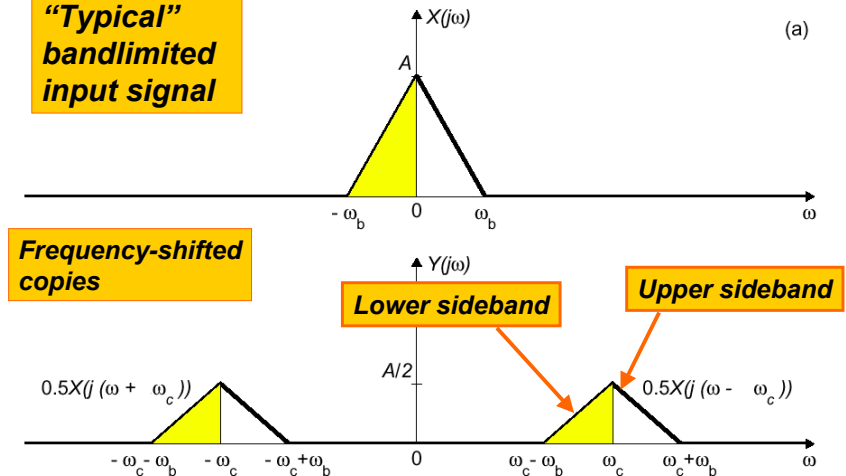
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Double Sideband AM (DSBAM)

“Typical”
bandlimited
input signal

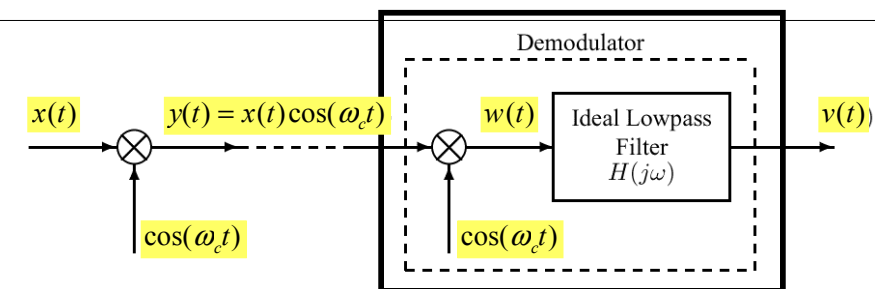


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DSBAM Demodulator



$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

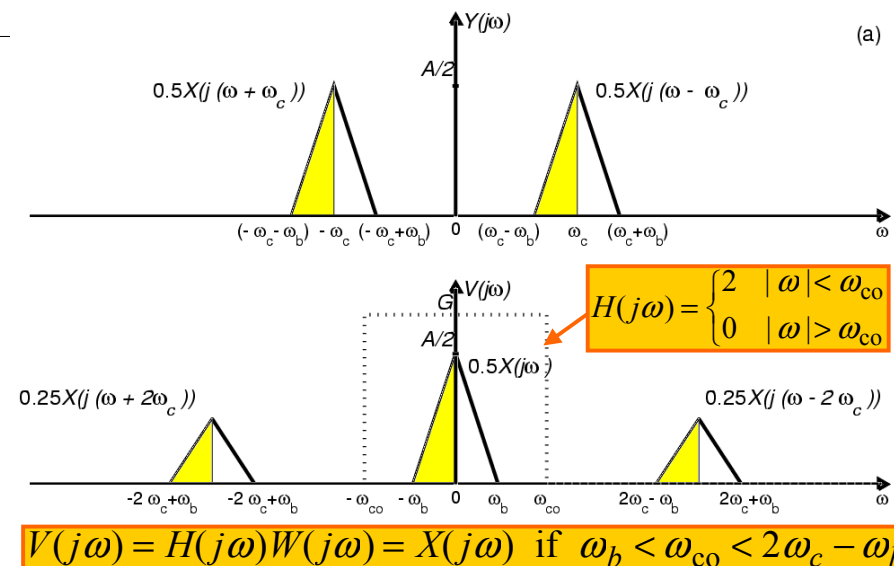
$$V(j\omega) = H(j\omega)W(j\omega)$$

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DSBAM Demodulation



$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

Frequency-Division Multiplexing (FDM)

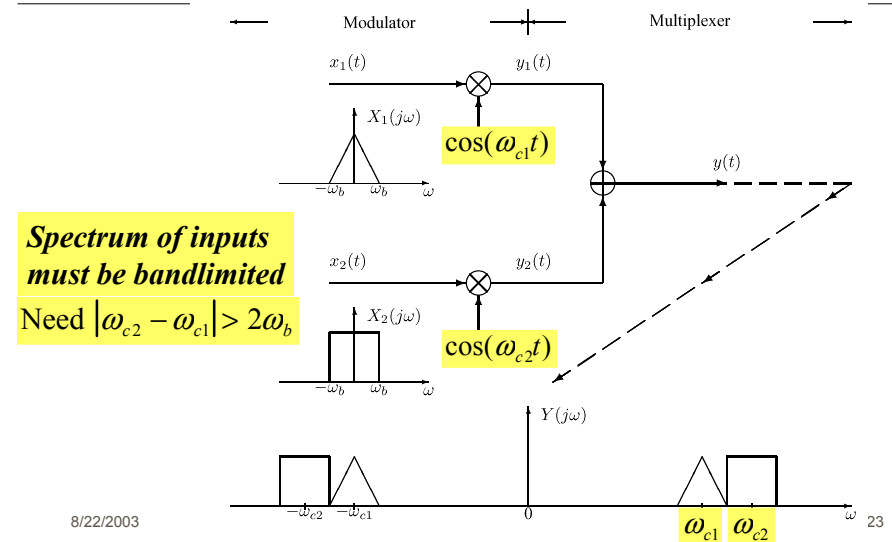
- Shifting spectrum of signal to higher frequency:
 - Permits transmission of low-frequency signals with high-frequency EM waves
 - By allocating a frequency band to each signal multiple **bandlimited** signals can share the same channel
 - AM radio: 530-1620 kHz (10 kHz bands)
 - FM radio: 88.1-107.9 MHz (200 kHz bands)

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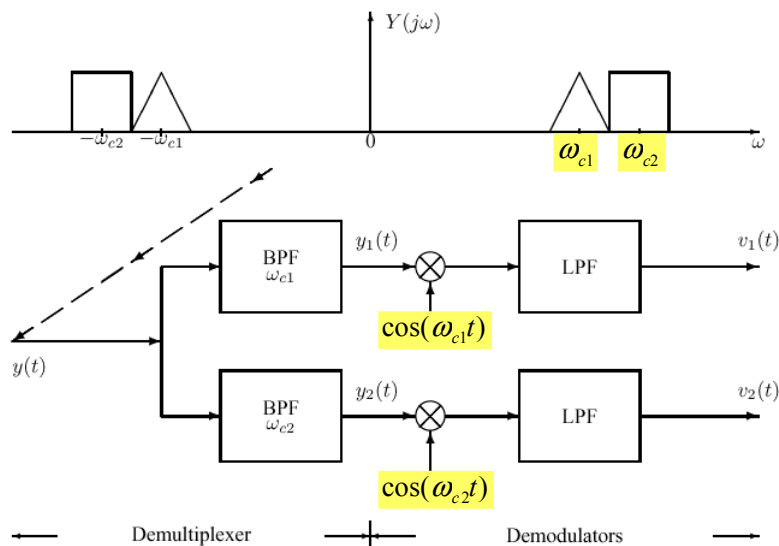
FDM Block Diagram (Xmitter)



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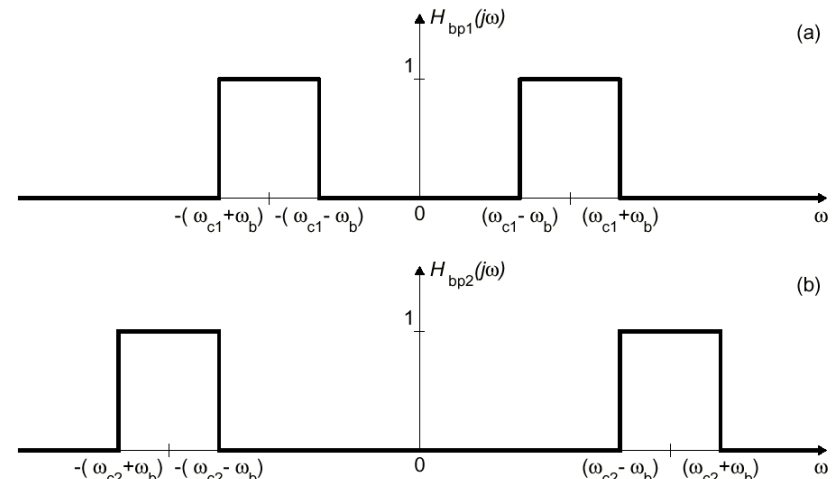
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Frequency-Division De-Mux

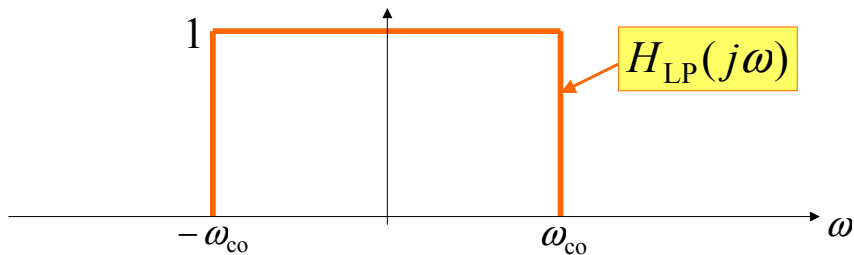


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Bandpass Filters for De-Mux



Pop Quiz: FT thru LPF



$$\text{Input } x(t) \leftrightarrow X(j\omega) = \sum_{k=-\infty}^{\infty} 4\pi\delta(\omega - 30\pi k)$$

If the output is $y(t) = 2$, then find a value for ω_{co}