PROBLEM 10.2:

$$H(j\omega) = \frac{3-j\omega}{3+j\omega} e^{-j\omega}$$

(a) $|H(j\omega)|^2 = H(j\omega) H^*(j\omega) = \frac{3-j\omega}{3+j\omega} e^{-j\omega} \frac{3+j\omega}{3-j\omega} e^{j\omega}$

$\Rightarrow |H(j\omega)|^2 = 1$ for all ω

(b) $\angle H(j\omega) = \angle\text{Numerator} - \angle\text{Denominator} - \omega$

$= \text{Tan}^{-1}\left(\frac{-\omega}{3}\right) - \text{Tan}^{-1}\left(\frac{\omega}{3}\right) - \omega$ from $e^{-j\omega}$

(c) $x(t) = 4 + \cos(3t)$

There are two freqs in $x(t)$: 0 and 3 rad/s

$\Rightarrow$ Evaluate $H(j\omega)$ at $\omega = 0$ and $\omega = 3$

$$H(j0) = \frac{3-j0}{3+j0} e^{-j0} = 1$$

$H(j3)$ has a magnitude of 1 (from part (a))

$\angle H(j3) = \text{Tan}^{-1}\left(\frac{-3}{3}\right) - \text{Tan}^{-1}\left(\frac{3}{3}\right) - 3$ (from part (b))

$= -\pi/4 - (\pi/4) - 3$

$= -\pi/2 - 3 \approx -4.571$

If we add 2π, the phase becomes $\angle H(j3) = 1.712$

$y(t) = 4 \cdot H(j0) + |H(j3)| \cos(3t + \angle H(j3))$

$= 4 + \cos(3t + 1.712)$

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PROBLEM 10.5:

(a) The period is $T_0 = 8$, so $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s

$$a_k = \frac{1}{8}\int_{-1}^{1} 10\, e^{-j\frac{\pi}{4}kt}\,dt$$

The limits on the integral are NOT -4 to $+4$ because $x(t)$ is ZERO for $-4 \le t < -1$ and $1 < t \le 4$.

(b) To plot the spectrum, we need the values of a_k for $k = -4, -3, -2, -1, 0, 1, 2, 3, 4$.

At $k=0$ use L'Hôpital's rule or take $\lim_{k\to 0}$

$$a_0 = \frac{10\,(\pi k/4)}{\pi k} = \frac{10}{4} = 2.5$$

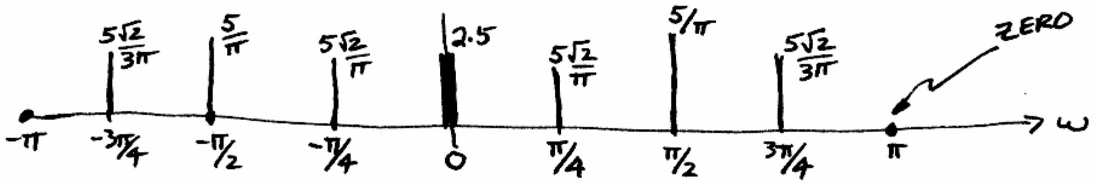
$$a_1 = \frac{10\sin(\pi/4)}{\pi} = \frac{10\cdot\sqrt{2}/2}{\pi} = \frac{5\sqrt{2}}{\pi}$$

NOTE: $a_{-1} = a_1$ and generally we have $a_k = a_{-k}$

$$a_2 = \frac{10\sin(\pi/2)}{2\pi} = \frac{10}{2\pi} = \frac{5}{\pi} = a_{-2}$$

$$a_4 = \frac{10\sin(\pi)}{\pi k} = 0$$

$$a_3 = \frac{10\sin(3\pi/4)}{3\pi} = \frac{10\sqrt{2}/2}{3\pi} = \frac{5\sqrt{2}}{3\pi} = a_{-3}$$

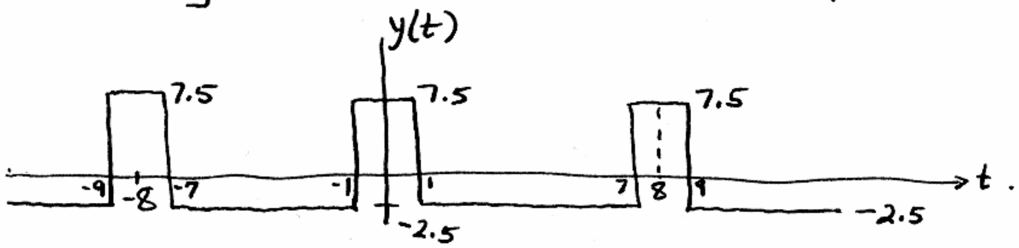


(c) The frequency response of the filter will MULTIPLY the spectrum of the input. Thus the spectrum of the output will be everything EXCEPT the line at DC.

Thus $y(t)$ has a FOURIER Series that is identical to the FS for $x(t)$ except the a_0 term is missing

$$\Rightarrow \quad y(t) = x(t) - a_0 = x(t) - 2.5$$

Subtracting a constant will shift the plot down



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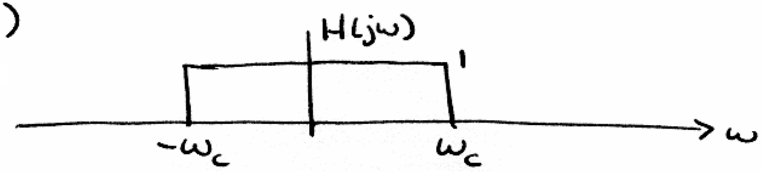
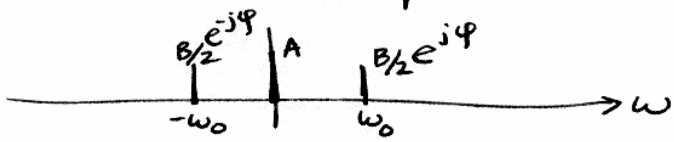
PROBLEM 10.5 (more):

(d)

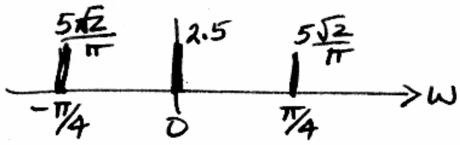


Again, we note that H(jω) will MULTIPLY the spectrum of x(t). We want the spectrum of the output to be



Since $\omega_o = \pi/4$, we need $\omega_c > \omega_o$. But we also need $\omega_c < 2\omega_o$. Thus $\frac{\pi}{4} < \omega_c < \frac{\pi}{2}$

With this ω_c, the Spectrum of y(t) will be:



$\Rightarrow$

$$y(t) = 2.5 + \frac{10\sqrt{2}}{\pi}\cos\left(\frac{\pi}{4}t\right)$$

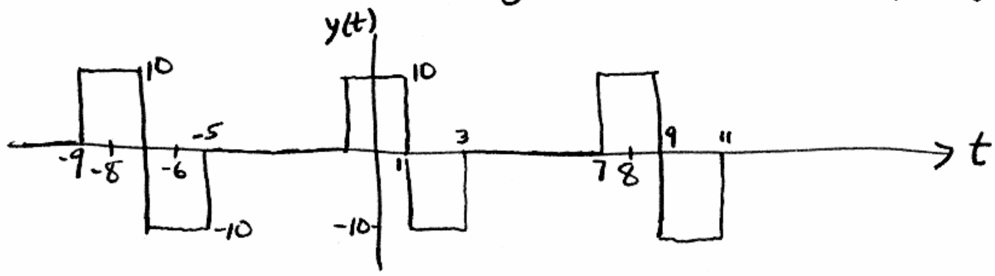
(e) If $H(j\omega) = 1 - e^{-j2\omega}$ we can find $h(t)$ by doing an INVERSE FOURIER TRANSFORM.

$$h(t) = \delta(t) - \delta(t-2)$$

Then $y(t) = x(t) * h(t) = x(t) * \delta(t) - x(t) * \delta(t-2)$

$= x(t) - x(t-2)$.

So we must shift x(t) by 2 and then subtract



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PROBLEM 10.8:

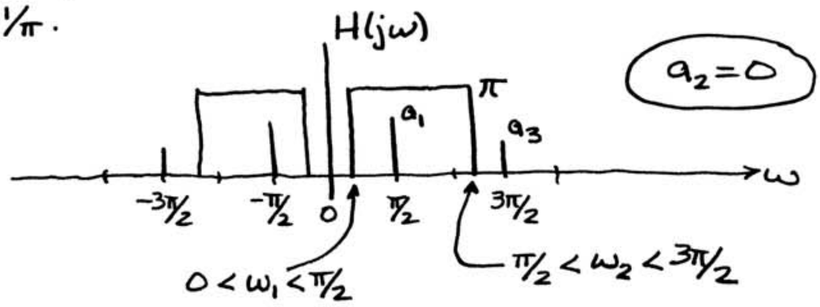
(a) $\omega_0 = 2\pi/T_0 = 2\pi/4 = \pi/2$ rad/s

(b) The Fourier Series coefficients for the 50% duty cycle square wave were derived in Chapter 3

$$a_k = \begin{cases} \frac{1}{2} & k=0 \\ 0 & k=\pm 2, \pm 4, \pm 6, \ldots \\ \frac{\sin(\pi k/2)}{\pi k} & k = \pm 1, \pm 3, \pm 5, \ldots \end{cases}$$

(c) $y(t) = 2\cos\left(\frac{2\pi t}{4}\right) = 2\cos\left(\frac{\pi}{2} t\right)$

Since the frequency of $y(t)$ is $\pi/2$ which is ω_0 the filter just needs to pass a_1 & a_{-1}. Also, the gain of the BPF needs to be π because $|a_1| = 1/\pi$.



$$H(j\omega) = \begin{cases} 0 & |\omega| < \omega_1 \\ \pi & \omega_1 \le |\omega| \le \omega_2 \\ 0 & \omega_2 \le |\omega| \end{cases}$$

(d) $y(t) = 2\cos\left(\frac{2\pi}{3} t\right)$

The frequency of $y(t)$ is $\frac{2\pi}{3}$ rad/s which is NOT an integer multiple of $\omega_0 = \frac{\pi}{2}$. Hence, there is no LTI system that will have $y(t)$ as its output when the square wave $x(t)$ is the input.

PROBLEM 11.4:

(a) $x(t) = u(t) - u(t-4)$ is a shifted pulse

$$= \underset{\text{time-shift}}{\delta(t-2)} * \underbrace{[u(t+2) - u(t-2)]}_{\text{F.T.} = \frac{\sin(2\omega)}{\omega/2}}$$

$$X(j\omega) = e^{-j2\omega}\,\frac{\sin(2\omega)}{\omega/2}$$

(b) Each impulse in ω inverts to a complex exponential

$$S(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) + 2\pi\delta(\omega + 10\pi)$$

$$s(t) = 2e^{j0} + e^{j10\pi t} + e^{-j10\pi t}$$

$$= 2 + 2\cos(10\pi t)$$

(c) $R(j\omega) = \frac{1}{2} - \frac{2}{4 + j2\omega} = \frac{1}{2} - \frac{1}{2 + j\omega}$

$$r(t) = \tfrac{1}{2}\delta(t) - e^{-2t}u(t)$$

(d) $y(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$$Y(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos(\omega)$$

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PROBLEM 11.9:

$h(t)$ is real $\Rightarrow$ $h^*(t) = h(t)$

If $h(t) \longrightarrow H(j\omega)$, then $h^*(t) \longrightarrow H^*(-j\omega)$

$\Rightarrow$ $H^*(-j\omega) = H(j\omega)$

Express $H(j\omega)$ in terms of its real and imaginary parts:

$$H(j\omega) = A(\omega) + j\,B(\omega)$$

Then $H^*(-j\omega) = A(-\omega) - j\,B(-\omega)$

$\Rightarrow$ $A(\omega) = A(-\omega)$ and $-B(\omega) = B(-\omega)$

The magnitude is even:

$$|H(-j\omega)| = \sqrt{A^2(-\omega) + B^2(-\omega)}$$
$$= \sqrt{A^2(\omega) + B^2(\omega)} = |H(j\omega)|$$

The phase is odd:

$$\angle H(-j\omega) = \operatorname{Tan}^{-1}\left\{\frac{B(-\omega)}{A(-\omega)}\right\}$$
$$= \operatorname{Tan}^{-1}\left\{\frac{-B(\omega)}{A(\omega)}\right\}$$
$$= -\operatorname{Tan}^{-1}\left\{\frac{B(\omega)}{A(\omega)}\right\} = -\angle H(j\omega)$$

Recall that the tangent function is an <u>ODD</u> function.

PROBLEM 11.12:

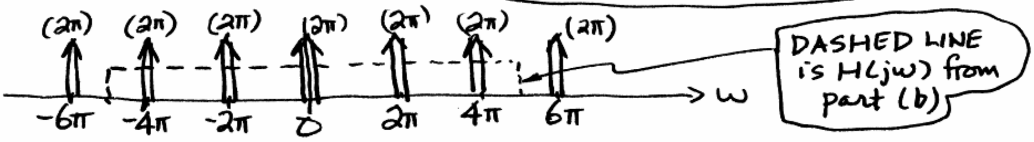
(a) The F.T. of an impulse train is another impulse train.

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) = \sum_{k=-\infty}^{\infty} e^{j2\pi kt}$$

Each complex exp transforms to $2\pi\delta(\omega - k\omega_0)$

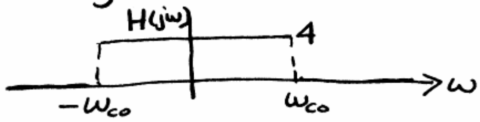
$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

NOTE: $\omega_0 = 2\pi$ rad/s



(b) The F.T. of a "sinc" is a rectangle

$$h(t) = 4\,\frac{\sin(\omega_{co}t)}{\pi t} \longrightarrow$$



(c) $Y(j\omega)$ will consist of 5 impulses because $H(j\omega)=0$ when $|\omega| > 5\pi$

$$Y(j\omega) = 8\pi\delta(\omega) + 8\pi\delta(\omega-2\pi) + 8\pi\delta(\omega+2\pi) + 8\pi\delta(\omega-4\pi) + 8\pi\delta(\omega+4\pi)$$

Each $\delta(\omega - ?)$ will inverse F.T. to an exponential.

$$y(t) = 4 + 4e^{j2\pi t} + 4e^{-j2\pi t} + 4e^{j4\pi t} + 4e^{-j4\pi t}$$

$$y(t) = 4 + 8\cos(2\pi t) + 8\cos(4\pi t) \quad \text{for } \underline{\text{all}}\ t$$

(d) If you want only the constant term then put the cutoff frequency of the filter below 2π rad/s, but keep it greater than ZERO.

$$\boxed{0 < \omega_{co} < 2\pi}$$

In this case, $Y(j\omega) = 8\pi\delta(\omega)$

$$\Rightarrow y(t) = 4 \qquad \boxed{c = 4}$$

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PROBLEM 11.17:

Define $s(t) = u(t) - \frac{1}{2}$

$$\Rightarrow \quad S(j\omega) = U(j\omega) - \pi\delta(\omega)$$

Since $s(t)$ is an odd function: $s(-t) = -s(t)$

and $s(-t) \xrightarrow{FT} S(-j\omega)$

$$S(-j\omega) = -S(j\omega)$$

$$\Rightarrow \quad U(-j\omega) \underbrace{-\pi\delta(-\omega)}_{=\pi\delta(\omega)} = -U(j\omega) + \pi\delta(\omega)$$

$$U(-j\omega) = -U(j\omega) + 2\pi\delta(\omega)$$

Thus, if we assume $U(j\omega) = \frac{1}{j\omega} + K\delta(\omega)$

$$\frac{1}{-j\omega} + K\delta(-\omega) = -\frac{1}{j\omega} - K\delta(\omega) + 2\pi\delta(\omega)$$

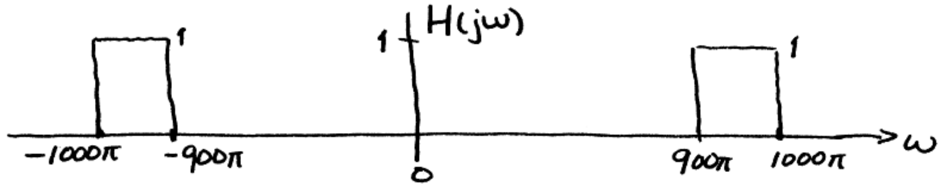
$$\Rightarrow \quad 2K\delta(\omega) = 2\pi\delta(\omega)$$

$$\Rightarrow \quad K = \pi$$

Note: $\delta(-\omega) = \delta(\omega)$, i.e. $\delta(\,)$ is <u>even</u>

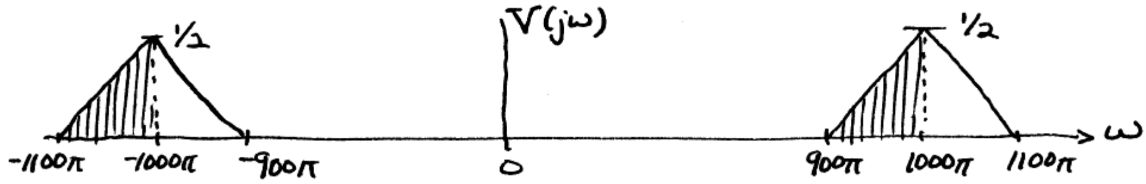
PROBLEM 12.4:

(a)

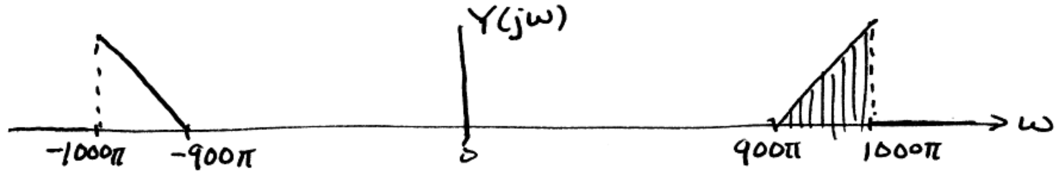


(b) Multiplying by a cosine will cause "frequency shifting"

$$V(j\omega) = \frac{1}{2} X(j(\omega + 1000\pi)) + \frac{1}{2} X(j(\omega - 1000\pi))$$



(c) Multiply the two graphs



(d) The term "single sideband" (SSB) comes from the following observations:

1. The original $X(j\omega)$ has a positive frequency portion and a negative frequency portion (shaded)
2. In $V(j\omega)$ the shaded portion is called the lower sideband because it is below the "carrier frequency" of 1000π rad/s in this case.
3. The final spectrum (i.e., F.T.) for $Y(j\omega)$ has only one of the sidebands for $w>0$ and the other one for $w<0$. Thus a single sideband remains

PROBLEM 12.7:

a)

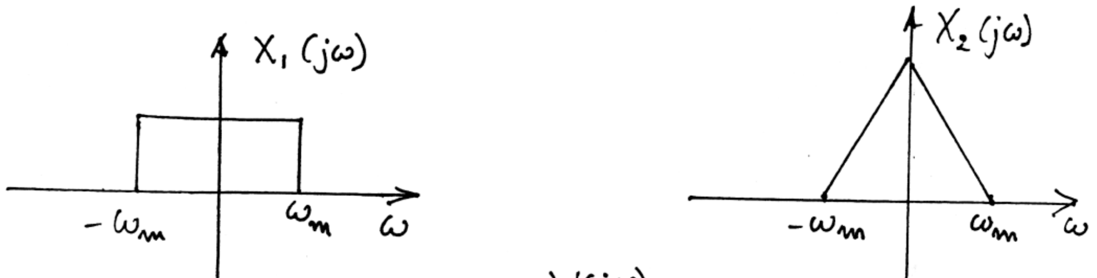
$$x_1(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X_1[j(\omega-\omega_c)] + \frac{1}{2}X_1[j(\omega+\omega_c)]$$

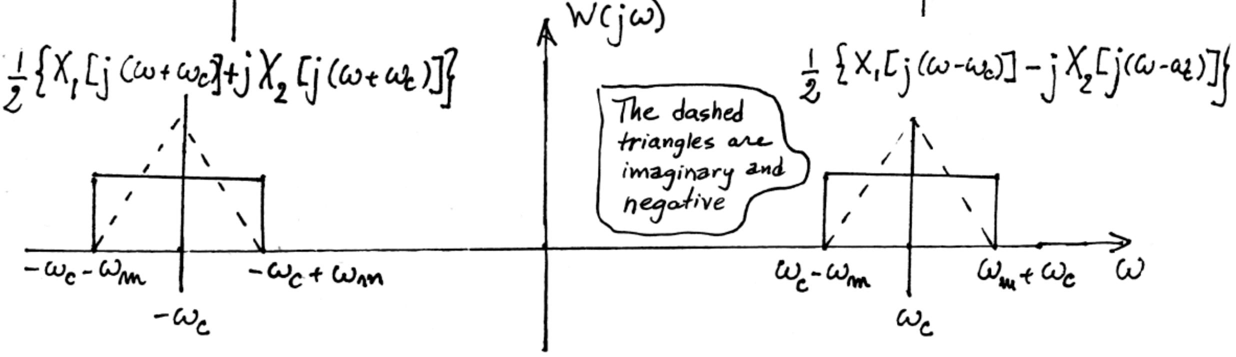
$$x_2(t)\sin(\omega_c t) \longleftrightarrow \frac{1}{2j}X_2[j(\omega-\omega_c)] - \frac{1}{2j}X_2[j(\omega+\omega_c)]$$

$$w(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t) \longleftrightarrow$$

$$W(j\omega) = \frac{1}{2}\left\{X_1[j(\omega-\omega_c)] - jX_2[j(\omega-\omega_c)]\right\} +$$

$$+\frac{1}{2}\left\{X_1[j(\omega+\omega_c)] + jX_2[j(\omega+\omega_c)]\right\}$$





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PROBLEM 12.7 (more):

b) $\omega_a = \omega_c - \omega_m$

$\omega_b = \omega_c + \omega_m$

c) $v(t) = w(t)\sin(\omega_c t) = x_1(t)\sin(\omega_c t)\cos(\omega_c t) +$

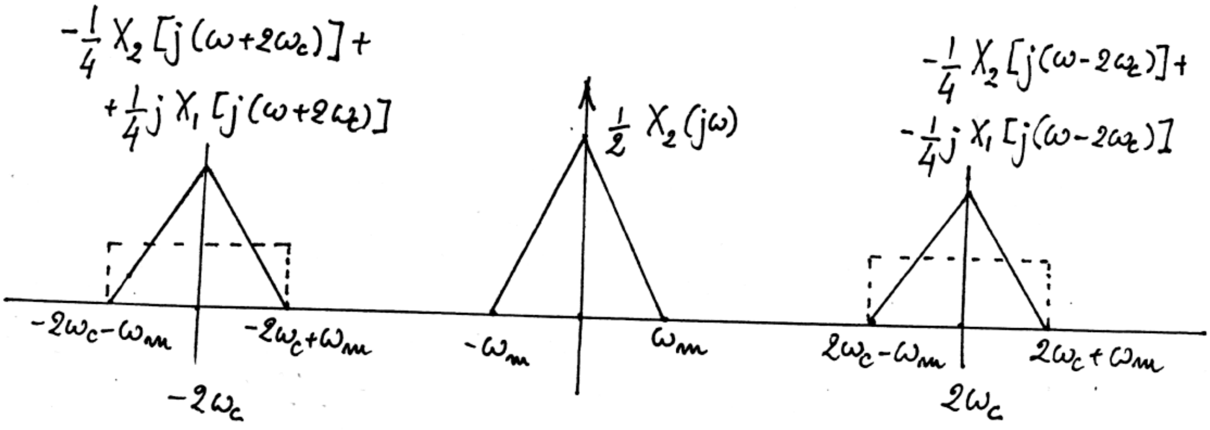
$+ x_2(t)\sin^2(\omega_c t) = \frac{1}{2}x_1(t)\sin(2\omega_c t) +$

$+ \frac{1}{2}x_2(t)[1 - \cos(2\omega_c t)] = \frac{1}{2}x_2(t) +$

$+ \frac{1}{2}x_1(t)\sin(2\omega_c t) - \frac{1}{2}x_2(t)\cos(2\omega_c t)$

$V(j\omega) = \frac{1}{2}X_2[j\omega] + \frac{1}{4j}\{X_1[j(\omega - 2\omega_c)] +$

$- X_1[j(\omega + 2\omega_c)]\} - \frac{1}{4}\{X_2[j(\omega - 2\omega_c)] + X_2[j(\omega + 2\omega_c)]\}$



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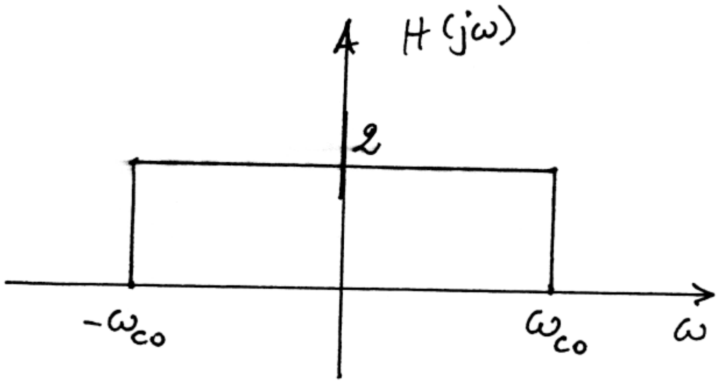
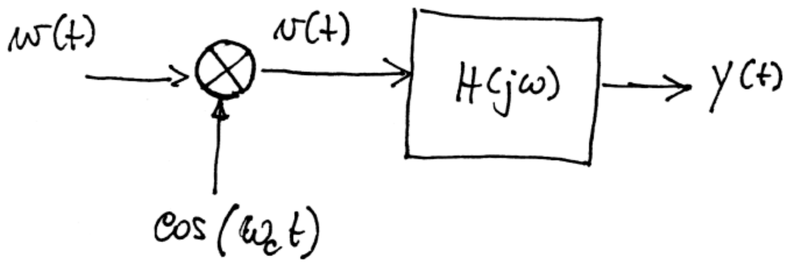
PROBLEM 12.7 (more):

d)



Ideal low-pass filter with gain equal to 2 and $\omega_m \leq \omega_{co} \leq 2\omega_c - \omega_m$ (in fact, a non-ideal low-pass filter will also work).

e)



$$v(t) = x_1(t)\cos^2(\omega_c t) + x_2(t)\sin(\omega_c t)\cos(\omega_c t) =$$

$$= \frac{1}{2}x_1(t)\left[1 + \cos(2\omega_c t)\right] + \frac{1}{2}x_2(t)\sin(2\omega_c t) =$$

$$= \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos(2\omega_c t) + \frac{1}{2}x_2(t)\sin(2\omega_c t)$$

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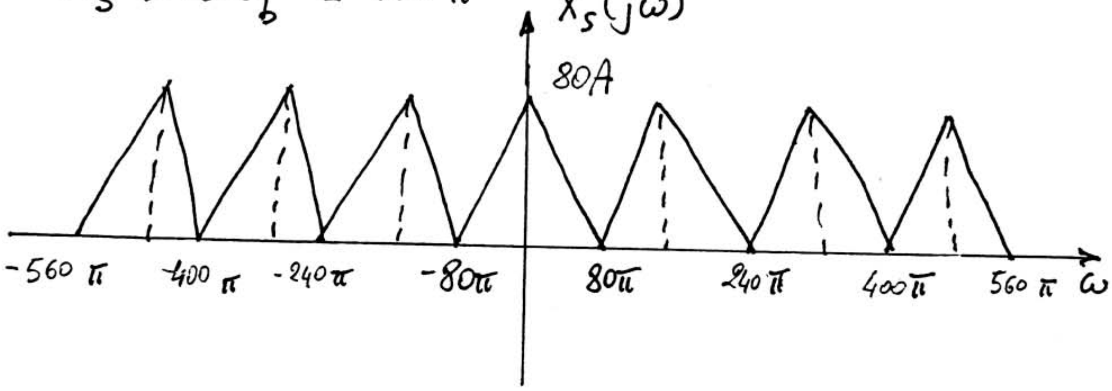
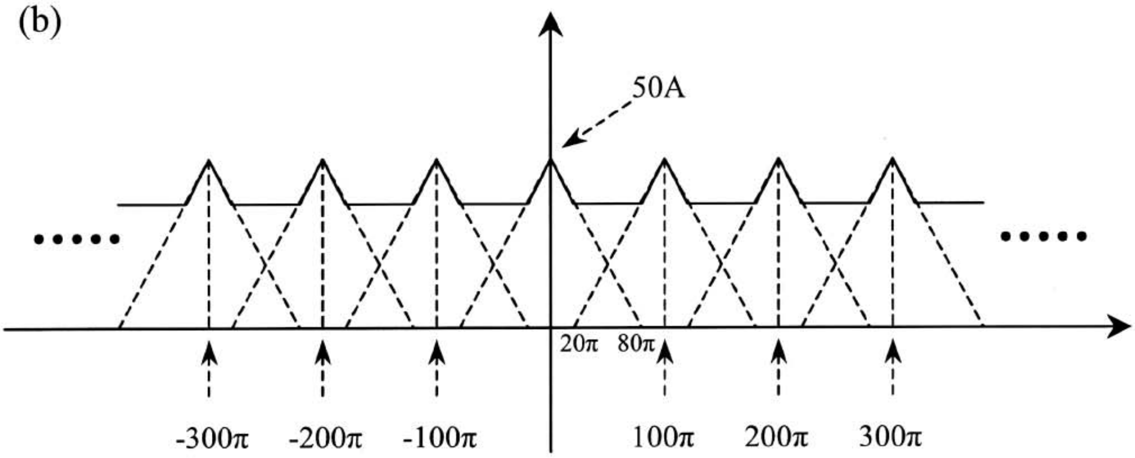
PROBLEM 12.12:

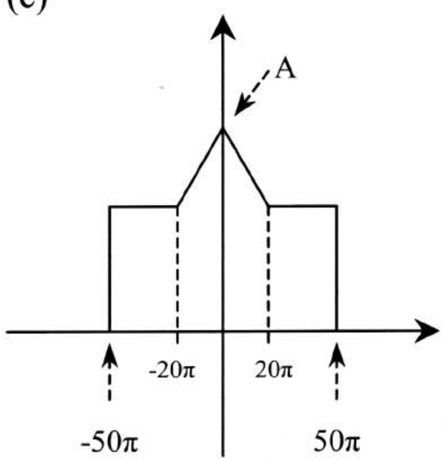
a) $\omega_s \geq 2\omega_b = 160\pi$



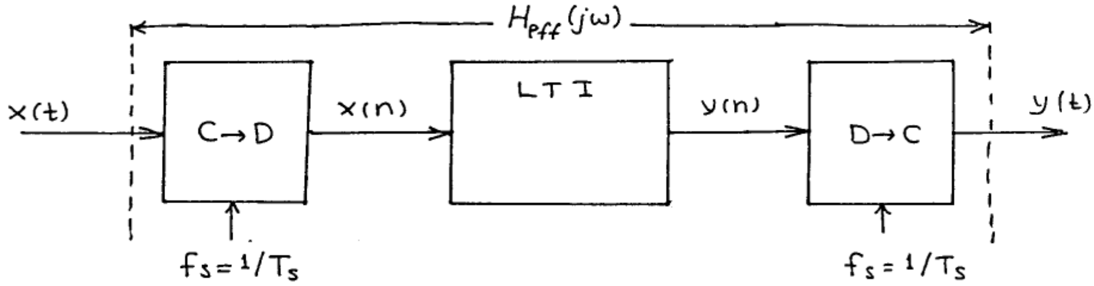
(b)



(c)



PROBLEM 12.15:



(a) $y(n) = 0.8\,y(n-1) + x(n) + x(n-2)$

$f_s = 200$ Hz

$$Y(z) = 0.8z^{-1}Y(z) + X(z) + z^{-2}X(z) \Rightarrow$$

$$H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}$$

$$H(e^{j\hat{\omega}}) = \frac{1 + e^{-j2\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}} \qquad -\pi < \hat{\omega} < \pi$$

$$H_{eff}(j\omega) = \frac{1 + e^{-j2(\omega/200)}}{1 - 0.8e^{-j(\omega/200)}} \qquad -\pi \cdot 200 < \omega < \pi\, 200$$

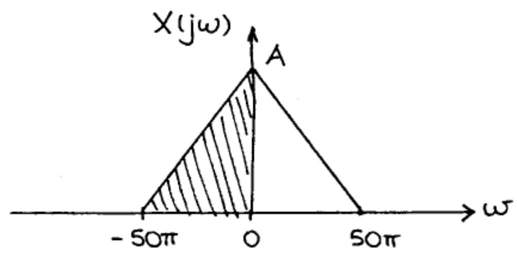
$$\hat{\omega} = 2\pi \hat{f} = 2\pi \frac{f}{f_s} = \omega T_s = \omega/200$$

$$y(t) = 2\,|H_{eff}(j(\omega = 100\pi))|\,\cos(100\pi t + \angle H_{eff}(\omega = 100\pi))$$

$$H_{eff}(\omega = 100\pi) = \frac{1 + e^{-j2\frac{100\pi}{200}}}{1 - 0.8e^{-j\frac{100\pi}{200}}} = \frac{1 + e^{-j\pi}}{1 - 0.8e^{-j\pi/2}} = 0$$

Therefore $y(t) = 0$

(b)



$f_s \geq 2 f_{max}$

$\omega_{max} = 50\pi = 2\pi(25) \rightarrow$

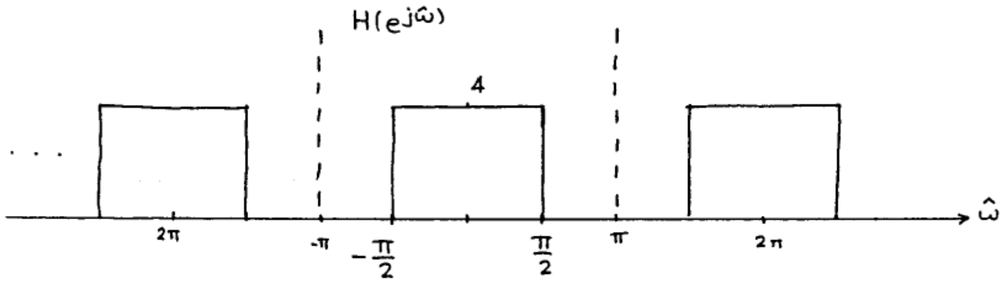
$f_{max} = 25$ Hz

$f_s \geq 2 \cdot 25 = 50$ Hz $\rightarrow$

$f_{s\,min} = 50$ Hz

PROBLEM 12.15 (more):

(c)

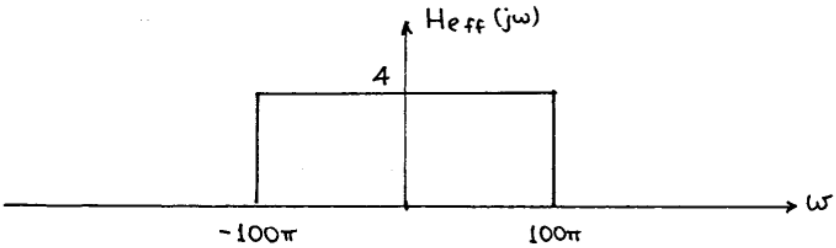


$f_s = 200\,\text{Hz}$

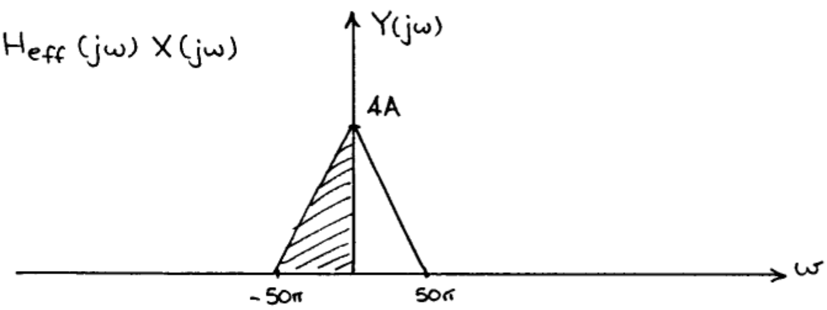
$$H_{eff}(j\omega) = H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\omega T_s} = H(e^{j\omega/200}) \qquad -\frac{\pi}{2} < \omega T_s = \hat{\omega} < \frac{\pi}{2}$$

$$\rightsquigarrow \quad H_{eff}(j\omega) = \begin{cases} 4 & \left|\frac{\omega}{200}\right| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \quad \rightsquigarrow$$

$$H_{eff}(j\omega) = \begin{cases} 4 & |\omega| < 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$



$$Y(j\omega) = H_{eff}(j\omega)\,X(j\omega)$$



(d) $\quad -\frac{\pi}{2} < \hat{\omega} < \frac{\pi}{2} \quad \rightsquigarrow \quad -\frac{\pi}{2} < \omega T_s < \frac{\pi}{2} \quad \rightarrow \quad -\frac{\pi}{2} f_s < \omega < \frac{\pi}{2} f_s$

For $X(j\omega) = Y(j\omega) \rightsquigarrow \quad 50\pi \leq \frac{\pi}{2} f_s \Rightarrow f_{s\,min} = \frac{100\pi}{\pi} = 100\,\text{Hz}$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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